Universität Kalrsruhe (TH)

Institut für Algebra und Geometrie

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In WS2006/07 I will teach a course on

Modular Forms

This theme links (complex) analysis, geometry, algebra, and number theory. The overall feature is that number theoretic questions lead to analytic functions who can be analyzed with the help of geometric and algebraic tools. Our main language will be that of complex analysis.

Beginning with basic properties of doubly periodic meromorphic functions we will study the Weierstraß-$\wp$-function and its differential equation. This leads to Eisensteinseries, which are holomorphic functions on the upper halfplane $\mathbb{H} := \{z \in \mathbb{C} \mid \mathrm{Im}(z) > 0\}$. We will study the geometry of this halfplane (with a certain natural metric) and thereby find a modell of plane hyperbolic geometry.

The Eisensteinseries behave nicely with respect to certain isometries of the hyperbolic plane (Möbius transformations with matrices in $\mathrm{SL}(2, \mathbb{Z})$: the elliptic modular group). It is this kind of behaviour which defines modular forms.

We will see all modular forms for $\mathrm{SL}(2, \mathbb{Z})$ (which form a ring generated by the Eisensteinseries) and will then turn to study so-called Hecke operators. They are diagonalisable, and their eigenvalues are closely related to the Fourier coefficients of the eigenfunctions.

These Fourier coefficients are of numbertheoretical interest and have been the driving force in studying modularforms since more than 150 years. This begins with studying multiplicities of quadratic forms (which will not be a topic in our lecture) and covers partition functions and many special functions in number theory. A deep conceptual understanding of modular forms and their Fourier coefficients is one of the main ingrediences in Wiles' proof of Fermat's Last Theorem (however, I will not say much about this).

Audience: Everybody who is willing to believe the Residue Theorem in english language. In particular, I do invite students of our International Programme, all students of mathematics from Fachsemester 5 upwards, physicists (I won't say anything on particle physics and moduli spaces, though) and Lehramtsstudierende.

Prerequisites: Complex analysis (i.e. residue theorem), a tiny bit of algebra (groups and subgroups), and linear algebra (spectral theorem).

Co-ordinates: The lecture is scheduled to take place on tuesdays from 9:45h to 11:15h and fridays from 11:30h to 13:00h in room S12. There will not be any problem sheets or exercise sessions. However, I tend to include lots and lots of examples in my lectures.

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