Connectivity of arc complexes - Part II

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Next week we will continue to study different kinds of arc complexes and show that they are highly connected.

Connectivity of joins

Let K and L be two simplicial complexes with vertices K_0 and L_0, respectively. The join $K \star L$ of K and L is the simplicial complex with vertices $K_0 \sqcup L_0$ and

$$K \star L = \left\{ \sigma = < a_0, \ldots, a_k, b_0, \ldots, b_l > \mid \begin{array}{l} < a_0, \ldots, a_k > \in K \text{ and} \\ < b_0, \ldots, b_l > \in L \end{array} \right\},$$

i.e. it is the smallest complex which contains for each simplex $\sigma_1 = < a_0, \ldots, a_k >$ in K and each simplex $< b_0, \ldots, b_l >$ in L the simplex $< a_0, \ldots, a_k, b_0, \ldots, b_l >$.

Suppose that K is contractible. What can you say about $K \star L$?
More generally, what can you say about the connectivity of $K \star L$, if K is c_1-connected and L is c_2-connected?

A special arc complex

Let S be a surface of genus 1 with one boundary component and p a point on the boundary, $\Delta = \{p\}$. Consider the arc complex $A(S, \Delta)$. How does it look like? Do you recognise it from other occasions as e.g. elementary number theory or elementary geometry?