Next week we will continue to study different kinds of arc complexes and show that they are highly connected.

**Connectivity of joins**

Let $K$ and $L$ be two simplicial complexes with vertices $K_0$ and $L_0$, respectively. The join $K \ast L$ of $K$ and $L$ is the simplicial complex with vertices $K_0 \sqcup L_0$ and

$$K \ast L = \left\{ \sigma = <a_0, \ldots, a_k, b_0, \ldots, b_l> | <a_0, \ldots, a_k> \in K \text{ and } <b_0, \ldots, b_l> \in L \right\},$$

i.e. it is the smallest complex which contains for each simplex $\sigma_1 = <a_0, \ldots, a_k>$ in $K$ and each simplex $<b_0, \ldots, b_l>$ in $L$ the simplex $<a_0, \ldots, a_k, b_0, \ldots, b_l>$.

Suppose that $K$ is contractible. What can you say about $K \ast L$?

More generally, what can you say about the connectivity of $K \ast L$, if $K$ is $c_1$-connected and $L$ is $c_2$-connected?

**A special arc complex**

Let $S$ be a surface of genus 1 with one boundary component and $p$ a point on the boundary, $\Delta = \{p\}$. Consider the arc complex $A(S, \Delta)$. How does it look like? Do you recognise it from other occasions as e.g. elementary number theory or elementary geometry?