In this seminar we consider the simplicial complex made of isotopy classes of arcs in a surface $S$, with their end points in the boundary of the surface.

Let $\Delta$ denote a finite set of points in the boundary of $S$. An arc in $S$ with its ends in $\Delta$ is said to be trivial if it separates the surface into two parts with one of them being a disk containing no other points in $\Delta$ than its end points. For example the arc in the following figure is trivial:

![Figure 1: The black circles are boundary components of the annulus; the green arc is trivial](image)

Now we have the following challenge problems:

1. **Simplicial complex of a disk**

Suppose the surface is a disk.

   (a) Please check that for $A(S, \Delta)$ to be non-empty, $\Delta$ has to contain at least four points in the boundary of the disk.
(b) What happens if \( \Delta \) contains 5 or 6 points? Which topological spaces are the two corresponding complexes? Can you guess which topological spaces the complexes for a disk are in general?

2. **An example of contractible arc complexes**

In this part we are going to learn a theorem which states that in general the arc complexes are contractible, except for some special cases (for example, \( S \) is a disk). Can you convince yourself of the contractibility for the arc complex corresponding to the surface shown in the following figure and \( \Delta = \{p_1, p_2, p_3\} \)?

![Figure 2: \( \Delta \) contains one point in each boundary component of the pants.](image-url)