Weihnachtsworkshop on Geometry and Number Theory

Karlsruhe, December 17 to 19, 2018

Abstracts

Sebastian Baader. *Geometric braid groups.*

A positive braid is a finite word with positive exponents in the standard generators of a braid group. We associate a finitely presented group to each positive braid, which gives rise to a link invariant: if two positive braids represent the same link in the 3-sphere, then the corresponding groups are isomorphic. Judging from their properties, one might suspect that these groups are fundamental groups of some sort of objects. However, as we will see, they are not determined by the fundamental group of the corresponding link complements, and vice versa.
This is in collaboration with Michael Loenne.

Sven Caspart. *Translation manifolds and their Singularities.*

In this talk we want to discuss translation manifolds, a higher dimensional analogue of translation surfaces. For the purpose of the talk a translation manifold is a manifold with a translation structure, i.e. the changes of coordinates are translations. By pulling back the Euclidean metric via charts we can endow it with a flat metric. The singularities of such a translation manifold are the points of its metric completion which are not locally isometric to Euclidean space. For a two-dimensional manifold we receive back the definition of a (possibly infinite) translation surface. A simple Christmas example can be constructed by glueing all faces of your cubic presents under the Christmas tree by a translation, which is the analogue of gluing edges of polygons for higher dimensions. The resulting gift manifold has a natural translation structure. The singularities of it are (a subset of) the edges and corners of the presents glued. In the talk we want to have a closer look at such singularities and what properties they have.

Mladen Dimitrov. *Uniform boundedness for the rational torsion of abelian 3-folds of Picard type.*

In 1969 Manin proved a uniform version of Serre's celebrated result on the openness of the Galois image in the automorphisms of the p-adic Tate module of any non-CM elliptic curve over a given number field. Recently in a series of papers Cadoret and Tamagawa established a definitive result regarding the uniform boundedness of the p-primary torsion for 1-dimensional abelian families. In a collaboration with D. Ramakrishnan we provide first evidence in higher dimension, in the case of abelian families parametrized by Picard modular surfaces over an imaginary quadratic field M. Namely, we establish a uniform bound for the p-primary torsion of principally polarized abelian 3-folds with multiplication by M, but without CM factors, subject to some rationality condition at the primes dividing the discriminant of M.

Max Horn. *Kac-Moody symmetric spaces.*

Kac-Moody groups can be thought of as infinite-dimensional generalizations of semisimple Lie groups. They can be understood (at least in the so-called two-spherical case) as amalgams of rank-2-subgroups such as SL_3 or $SL_2 \times SL_2$. With these descriptions, many problems about Kac-Moody groups can be reduced to questions about SL_2, SL_3 or other semi-simple Lie groups of rank at most 2.
We will discuss what a symmetric space associated to a Lie group is. I will then give the definition for what we call "Kac-Moody symmetric space", and why we think that these are good and interesting analogues for classical symmetric spaces, despite the absence of a manifold structure. I will also state some recent results on these spaces, obtained jointly with Walter Freyn, Tobias Hartnick and Ralf Köhl (see https://arxiv.org/abs/1702.08426 for a preprint).

Steffen Kionke. *Profinite invariants of arithmetic groups.*

An arithmetic group is, roughly speaking, a group of matrices with integer entries; e.g. the special linear group $\mathrm{SL}(n, Z)$. In this talk we discuss properties of arithmetic groups that can be read off from the finite quotient groups - such properties are called "profinite". In particular, we will show that the sign of the Euler characteristic of an arithmetic group is profinite. A concise formula for the Euler characteristic of some arithmetic spin groups, which involves special values of L-functions, will be used to show that the actual value of the Euler characteristic is not a profinite invariant of arithmetic groups.
(This is based on joint work with H. Kammeyer, J. Raimbault and R. Sauer.)

Samuel Lelièvre. *Periodic billiard trajectories in the regular pentagon.*

We present an enumeration of periodic billiard trajectories in the regular pentagon. This enumeration is based on an analogue of the Farey or Stern-Brocot tree, adapted to the $(2, 5, \infty)$ triangle group (also known as Hecke(5) group). The golden ratio φ is key here, and a gcd algorithm for "golden integers"(elements in the ring $\mathbb{Z}[\varphi]$, the ring of integers in the number field $\mathbb{Q}(\varphi)$ or $\mathbb{Q}(\sqrt{5})$) appears.
Joint work with Diana Davis, with key use of SageMath and CoCalc.

Hannah Markwig. *Tropical curves and their moduli spaces.*

Tropical geometry can be viewed as a piecewise linear shadow of algebraic geometry. This emerging field of study, which allows a fruitful exchange of methods among algebraic geometry and discrete mathematics, has applications in particular in the theory of moduli spaces, enumerative geometry and mirror symmetry.
We introduce tropical curves and their moduli spaces and review basic properties, the relation to moduli spaces in algebraic geometry, important applications and latest developments in this area.

Verena Möhler. *On the full automorphism group of* $\mathrm{SL}(2, q)^\flat$*-unitals.*

Unitals are an interesting kind of incidence structure and $\mathrm{SL}(2, q)^\flat$-unitals a special construction of unitals, where the points outside one block at infinity are represented by the elements of $\mathrm{SL}(2, q)$.
We show that the full automorphism group of any $\mathrm{SL}(2, q)^\flat$-unital $\mathbb{U}^\flat_{\mathcal{D}}$ of order $q \geq 3$ fixes the block at infinity.

Scott Mullane. *Positivity of the zero residue strata of differentials.*
The strata of abelian differentials in the moduli space of pointed curves and their projections under forgetting marked points inform many aspects of the birational geometry of these moduli spaces when $g > 0$. This talk will focus on the special loci inside meromorphic strata that arise as a result of setting residues to zero. After presenting different ways such strata arise, we discuss the compactification of these strata and how they restrict to higher codimension boundary components. We then restrict to the case $g = 0$, where we show by an inductive argument that the divisors obtained by this construction are nef and lie on the boundary of the nef cone.

Jan-Christoph Schlage-Puchta. *Groups with many normal subgroups.*
Let Γ be a finitely generated group, $a_n^{\triangleleft}(\Gamma)$ be the number of normal subgroups of index n in Γ. If Γ is non-abelian free, then Lubotzky and Mann showed that $a_n(\Gamma)$ is of magnitude $n^{\log n}$. Later this estimate was extended to larger classes of groups. Here we give rather elementary results, which show that the class of groups which have about as many normal subgroups as a free non-abelian group is quite large, including all Golod-Shafarevic groups, and all groups for which $\Gamma/\gamma_2(\Gamma')$ is large. On the other hand we show that as a function of n, $a_n^{\triangleleft}(\Gamma)$ always behaves quite chaotic, and cannot be approximated by some smooth function in a very weak sense.

Christian Steinhart. *Explosions in Outer Space.* Culler-Vogtmann Space alias Outer Space CV_n can be seen as Teichmüller space for finite metric graphs without leaves. Analogue to the Thurston metric there exists an asymmetric Lipschitz-metric on CV_n which we will discuss during the talk. An interesting aspect is, that geodesics are almost never unique. By using the envelope of two points, that is the set of all geodesics from A to B, we will construct a geodesic, which is piecewise unique. Furthermore exploding envelopes determine the faces of reduced Outer Space, which implies that the isometry group of the reduced Outer Space is the isometry group of Outer Space.

Martin Ulirsch. *Tropical Brill-Noether theory and Prym varieties.*
Brill-Noether theory is a classical part of the geometry of Riemann surfaces that studies properties of special divisors, i.e. of divisors for which the correction term in the Riemann-Roch formula is positive. In recent years, a new approach to this story has emerged that is based on the geometry of special divisors on tropical curves. This combinatorial framework has already lead to new combinatorial proofs of some of the classical theorems of Brill-Noether theory as well as to several new previously unknown results.
In this talk, I will give an introduction to this very active area and explain the basic mechanisms that make this approach work. In particular, I will focus on geometric properties of the Prym-Brill-Noether locus, a generalization of the Brill-Noether locus in the Picard variety to the Prym variety of an unramified double cover. Our main result is a new tropical proof of the Prym-Brill-Noether Theorem for generic double covers (classically due to Welters) as well as a previously unknown upper bound on the dimension of the Prym-Brill-Noether locus for a generic double cover whose base has fixed even gonality.
This is joint work with Yoav Len (Georgia Institute of Technology).

Ferrán Valdez. *Around big Mapping Class Groups.*

Mapping class groups associated to surfaces whose fundamental group is not finitely generated are called "big" and its study is linked to classical problems in dynamics. In this talk we discuss some of the basic properties of big MCGs, their simplicial actions and how these can be used to prove that big MCGs "detect" surfaces or (if time allows) that the space of non-trivial quasimorphisms of a big mapping class group is infinite dimentional.

Christian Weiß. *Interval Exchange Transformations, Lyapunov Exponents and Discrepancy.*

Sequences with small discrepancy are a central ingredient in the Quasi-Monte Carlo method for efficient numerical approximation of integrals with deterministic error bounds. If their degree of uniformity is best possbile, these sequences are called low-discrepancy sequences. Among them Kronecker sequences build an important one-dimensional class. At the same time Kronecker sequences can be realized as oribts of circle rotations, which are in turn the simplest class of interval exchange transformations (IETs). It is natural to ask wheher there exist more general IETs which yield low-discrepancy sequences. In the case of three intervals, the criteria for circle rotations can easily be carried over. The dynamics of IETs with more than three intervals is much more complex. It was already pointed out in the late 1990s by Zorich that measuring the discrepancy of IET orbits is also closely connected to the Lyapunov exponents of the IET. However, his and other well-known abstract results can hardly be applied in practice and to our best knowledge did so far not allow any concrete examples of low-discrepancy orbits but circle rotations. In this talk, it is shown how to construct examples of IETs with an arbitrary number of intervals whose orbits are indeed low-discrepancy sequences.

Felix Wellen. *Abstract facts on formally étale maps.*

We use an abstract, category theoretic definition of formally étale maps which agrees with the usual definition on schemes of finite type over a field. We show properties of these maps, derive characterizations and give an overview of implications of these abstract results for other classes of maps.