# Weihnachtsworkshop on Geometry and Number Theory

## Karlsruhe, December 18 to 20, 2019

## Abstracts

**Tobias Columbus.** *Composition in Prederivators.*

Derivators are an axiomatisation of homotopy theories due independently to Franke, Grothendieck and Heller. In this talk, I will discuss a glueing lemma for certain diagrams in prederivators that leads to a notion of composition of coherent morphisms. If time permits, I will also touch upon potential applications.

**Vincent Delecroix.** *Dynamics of measured foliations on Riemann surfaces of infinite type.*

A quadratic differential $q$ on a Riemann surface $X$ gives rise to a one-parameter family of measured foliations. When $X$ is of finite type and $q$ has finite area such foliation is typically equidistributed w.r.t. the measure induced by the quadratic differential (Kerckhoff-Masur-Smillie theorem). For infinite type Riemann surface, many exotic examples can be constructed. I will describe some joint work with F. Valdez on the measurable completeness of these folations.

**Marvin Hahn.** *Mustafin models of plane curves and syzygy bundles.*

Mustafin varieties are degenerations of projective spaces, which are induced by point configurations in a Bruhat Tits building. In this talk, we use these degenerations to construct certain models of plane curves. Motivated by recent advances towards a $p$-adic Simpson correspondence, we then use these models to construct families of syzygy bundles which admit strongly semistable reduction. This talk is based on a joint work with Annette Werner.

**Christoph Karg.** *Watching infinite regular translation surfaces from very, very far.*

In the vast jungle of infinite translation surfaces we direct our attention to a highly interesting species: The family of regular translation surfaces. As we will see, we can obtain infinite regular translation surfaces by taking our favorite suitable Euclidean polygon and gluing infinitely many copies of it along edges in a symmetric way. Even though regular translation surfaces are not wild, they are definitely not harmless either: That's why we will only look at them from a large distance, i.e. we are interested in their large scale geometry. This talk also includes pictures of rare specimens!

**Shun Kumagai.** *A characterization of the Veech group of a flat structure with two finite Jenkins-Strebel directions.*

In this talk we deal with a sort of flat surfaces of which the Veech groups are characterized combinatorially. The cotangent bundle of the Teichmüller space can be regarded as a space of "flat structures" on surfaces (modulo isotopies). Affine deformations of a flat structure form an orbifold modeled on the disk in the moduli space and the group giving a representation of it is called the Veech group. When we assume flat surfaces to have two finite Jenkins-Strebel directions, we will see that the situation is similar to the one of "origamis". A relation between origamis and the absolute Galois group is shown by Möller.

**Marius Leonhardt.** *Plectic phenomena on Hilbert modular varieties.*

The modular curve has had various applications to number theory, in particular to the theory of elliptic curves. But what about applications of other Shimura varieties to higher dimensional abelian varieties? In this talk, we will focus on the Hilbert modular variety and explain one attempt to achieve such analogous applications, via the so-called "plectic conjecture". It involves a mysterious plectic Galois group and actions of this group on various quantities associated to the Hilbert modular variety. In particular, we will look at CM points and Galois actions on them.

**Frank Loose.** *Three-dimensional geometries without compact quotients.*

Thurston classified all 3-dimensional geometries with compact quotients building the cornerstones for his geometrization program for closed 3-manifolds. Motivated by cosmological models in general relativity we classified all 3-geometries not necessarily having compact models (joint with P. Konstantis).

**Oliver Lorscheid.** *Towards a cohomological understanding of the tropical Riemann Roch theorem.*

In this talk, we outline a program of developing a cohomological understanding of the tropical Riemann Roch theorem. We begin with a summary of the content and the significance of the tropical Riemann Roch theorem before we turn to a general outline of our program and its challenges. We continue with a more detailed discussion of some of those steps that have already been established. In particular, we will explain why blue schemes provide a suitable framework for tropical scheme theory. If time allows, we will explain the concept of a matroid bundle, which stems from a joint work with Matthew Baker.

**Duc-Manh Nguyen.** *Existence of periodic trajectories through a given point on translation surfaces.*

In this talk we will discuss the following question: given a regular point on a translation surface, does there always exist a periodic trajectory through this point? We will see how this question is related to the dynamics of the $\mathrm{GL}(2,\mathbb{R})$-action on moduli space of translation surfaces. Using the major breakthroughs of Eskin-Mirzakhani and Eskin-Mirzakhani-Mohammadi, together with recent results of Eskin-Filip-Wright and Apisa, we prove a dichotomy related to this problem, and provide an effective answer for some classes of translation surfaces. This is joint work with Huiping Pan and Weixu Su.

**Cecília Salgado.** *Mordell-Weil rank jumps and the Hilbert property.*

Let $X$ be an elliptic surface with a section defined over a number field. Specialization theorems by Néron and Silverman imply that the rank of the Mordell-Weil group of special fibers is at least equal to the MW rank of the generic fiber. We say that the rank jumps when the former is strictly larger than the latter. In this talk, I will discuss rank jumps for elliptic surfaces fibred over the projective line. If the surface admits a conic bundle we show that the subset of the line for which the rank jumps is not thin in the sense of Serre. This is joint work with Dan Loughran.

**Markus Stroppel.** *8, 7, 6, 5 — using octonions to understand classical groups.*

The classical groups (in particular, the simple algebraic groups over the complex num-

bers, or the simple Lie groups) come in infinite families, distinguished by type (e.g., linear/orthogonal/symplectic/hermitian) and parameterized by pairs $(V, f)$, where $V$ is a vector space of finite dimension over a suitable (not necessarily commutative) field $K$, and $f$ is a form of the given type on $V$. Roughly speaking, the type and the pair $(V, f)$ are determined by the isomorphism type of the group. However, there are some exceptional isomorphisms that cross these boundaries.
We use composition algebras (mainly of dimension 8) in order to better understand such exceptional isomorphisms in cases that are beyond the reach of the established theories of algebraic groups or of buildings.

**Andrea Thevis.** *Strata and Veech groups of p-origamis.*

We study a certain class of translation surfaces called $p$-origamis. These surfaces arise as normal covers of the torus with $p$-groups as deck transformation group. The goal is to classify the types of singularities of $p$-origamis and to show that these depend in most cases only on the isomorphism class of the deck transformation group. For this, we use the rich theory of $p$-groups. In the second part of the talk, I describe some results for Veech groups of $p$-origamis. Veech groups are finite index subgroups of $\mathrm{SL}(2, \mathbb{Z})$ and are related to the Teichmüller curves defined by the surfaces.

**Jonathan Zachhuber.** *Euler Characteristics of Strata.*

In joint work with Costantini and Möller, we use the recent smooth compactification of strata of abelian differentials by Bainbridge, Chen, Gendron, Grushevsky and Möller to calculate the (orbifold) Euler characteristic.

**Ferdinando Zanchetta.** *Operations on higher algebraic K-theory via unstable methods.*

Operations and algebraic structures on $K$-theory have been a very important and useful topic since the very beginning of $K$-theory itself (they were used to state and prove the Grothendieck-Riemann-Roch theorem, for example).
After some motivation drawing from both topology and algebraic geometry, we will introduce higher algebraic $K$-theory discussing briefly the approaches that can be taken in order to define operations and structures on it.
We will then focus on how we can use unstable homotopy theory to study this problem for a large class of possibly singular schemes, extending recent techniques coming from motivic homotopy theory.