Weihnachtsworkshop on Geometry and Number Theory

Karlsruhe, December 15 to 17, 2021

Abstracts

Simon Brandhorst. *Computation of the automorphism group of Enriques surfaces*
(Joint work with Ichiro Shimada.)

Enriques surfaces appear in an answer to a question of Castelnuovo (1895) on whether a complex surface with certain cohomological invariants is rational. Nowadays they are part of the classification of smooth projective complex surfaces with numerically trivial first Chern class.
After a brief introduction to Enriques surfaces we turn to their automorphism groups and how to compute them. Protagonists of this story are fractals and polyhedrons in hyperbolic space as well as Weyl groups.

Samantha Fairchild. *Analogs of the primitive Gauss circle problem*

Let $N(R)$ be the number of integer lattice points in the plane of length less than R. The expected value of $N(R)$ is roughly the area of the ball of radius R, $V(R)$. The Gauss circle problem is centered around the question of understanding the discrepancy, $D(R) = |N(R) - V(R)|$, which is the difference between the actual number of points and the expected number of points. A similar story can be told in the case of the *primitive* Gauss circle problem where instead of counting all integer lattice points, we restrict to lattice points whose entries are coprime. We will share expected value and discrepancy type results for counting discrete subsets which generalize the primitive integer lattice points.

Ursula Hamenstädt. *A boundary for the mapping class group*

We discuss the notion of a boundary for a group and why it is interesting. We then introduce a geometrically defined boundary for the mapping class group and discuss some applications.

Peter Kaiser. *Delone sets and the patch-counting function*

A Delone set is a uniformly discrete and relatively dense subset of a metric space. A patch is the configuration of points in a ball of given radius, the function which counts the number of equivalence classes of different configurations in dependence of the radius is the patch-counting function. This function measures how intricate the set is and we are especially interested in sets for which the patch-counting function is finite for all radii.
Recently Koivusalo and Walton gave us a way to determine the growth of the complexity function in the euclidean case for sets which arise from a well known construction. I will present this construction in the setting of locally compact second countable groups and also generalize their approach to this wider setup.

Manuel Kany, Pascal Kattler. *The question of arithmeticity for shadow Veech groups*

The affine orientation preserving homeomorphisms on a square tiled surface $\mathcal{O}$ define a representation on the homology $H_1(\mathcal{O}, \mathbb{Z})$. This representation respects a natural splitting. One of the corresponding subrepresentations which we call shadow Veech group will be the center of interest of this talk. We want to investigate its image and especially the question whether it is arithmetic.

Inder Kaur. *Birational geometry of blow-ups of projective spaces*

The Gale correspondence provides a duality between sets of n general points in projective spaces $\mathbb{P}^s$ and $\mathbb{P}^r$ when n equals $r+s+2$. By a result of Mukai, the blow-up of $\mathbb{P}^4$ at 8 points say X, can be realized as a moduli space of torsion-free rank 2 semi-stable sheaves (with certain fixed Chern class datum) on the blow-up of $\mathbb{P}^2$ in 8 Gale dual points. In a recent work, Casagrande, Codogni and Fanelli use this to describe the Mori chamber decomposition of the effective cone of divisors of X. It was shown by Castravet and Tevelev that the blow-up of $\mathbb{P}^r$ at n points for the case when $r \geq 5$ and $n \geq r+4$ is no longer a Mori dream space. In joint work with Carolina Araujo, Ana-Maria Castravet and Diletta Martinelli we show that even in this case it is possible to give a Mori chamber type decomposition for a part of the effective cone.

Steffen Kionke. *Amenability and profinite completions of finitely generated groups*

Is it possible to decide whether or not a finitely generated residually finite group is amenable by looking at its finite quotients only?
In this talk we answer this question in the negative and exhibit an uncountable family of counterexamples based on the theory of branch groups. On the other hand, we explain why uniform amenability, a stronger concept introduced in the 70's, can be detected from the finite quotients of residually finite groups.
This is based on joint work with Eduard Schesler.

Anja Randecker. *Trees everywhere (The saddle connection complex)*

A new approach to study translation surfaces is to encode their geometry in a combinatorial object, called the saddle connection complex. For translation surfaces, this complex plays the same role as its more established cousin, the arc complex, does for topological surfaces. I will define the saddle connection complex, introduce you to some of its properties, and - suitable for a Christmas workshop - we will see lots of trees (up to additive and multiplicative error).
The talk is based on joint works with Valentina Disarlo, Huiping Pan, and Robert Tang.

Fabian Ruoff. *The Moduli Space of Algebraic Translation Surfaces*

For the non-expert it might seem that the theory of translation surfaces is complex geometric in nature. Indeed, the first definition one usually encounters as gluing of planar polygons or as a complex manifold with translations as transition maps only really makes sense over $\mathbb{C}$. Things change when we characterize translation surfaces as pairs (X, ω) consisting of a compact Riemann surface together with a holomorphic one-form. As a compact Riemann

surface is nothing else than a complex curve this allows us to consider translation surfaces over different fields than $\mathbb{C}$. In this talk we show that the associated moduli space has the expected local properties, at least if the characteristic of the field under consideration is zero. Surprisingly, in positive characteristic there are some strata where this is no longer the case.

Jan-Christoph Schlage-Puchta. *Some additive problems involving powers of 2 and other thin sets*

Romanov showed that a positive proportion of all integers can be written as the sum of a prime and a power of two. Linnik showed that there is some K, such that every integer is the sum of two primes and at most K powers of 2. On the other hand Erdős and van der Corput proved independently that a positive proportion of all odd integers are not the sum of a prime and a power of 2. These results created quite some interest in additive problems involving primes and powers of 2.
In this talk we give an overview of additive problems involving sets with multiplicative structures and powers of 2 and other thin sets. We give rigorous and heuristic estimates for the density of sums of primes and a power of 2, prove a conjecture by Granville and Soundarajan on the sum of squarefree numbers and powers of 2, disprove a conjecture by Li and Pan on the sum of primes and arbitrary thin sequences, and construct surprisingly large sets of integers for which certain additive problems are unsolvable.
Parts of the talk are joint work with Elsholtz, Elsholtz and Planitzer, and Virchow.

Johannes Schwab. *Invariants of strata of k-differentials via intersection theory*

The Masur-Veech volume of (some) strata of quadratic differentials can be expressed as an intersection product by work of Chen, Möller and Sauvaget.
The Euler characteristic of a stratum of k-differentials can also be expressed as an intersection product on the compactification of the stratum with multi-scale k-differentials (joint work with Matteo Costantini and Martin Möller).
In this talk we present the latter formula and explain how to evaluate these intersection numbers.
The boundary divisors of the space of multi-scale k-differentials are indexed by coverings of so-called enhanced level graphs. An important ingredient to the evaluation is to determine which of those coverings in fact correspond to non-empty boundary strata. We give a purely combinatorial answer to this question (joint work with Felix Röhrle).

Davide Spriano. *Detecting hyperbolicity in* CAT(0) *spaces: from cube complexes to rank rigidity*

CAT(0) spaces form a classical and well-studied class of spaces exhibiting non-positive curvature behaviour. An important subclass of CAT(0) spaces are CAT(0) cube complexes, i.e. spaces obtained by gluing Euclidean n-cubes along faces, satisfying some additional combinatorial conditions. Given a CAT(0) cube complex, there are several techniques to construct spaces that "detect the hyperbolic behaviouröf the cube complex, but all of those techniques rely on the combinatorial structure coming from the cubes. In this talk we will present a new approach to construct such spaces that works for general CAT(0) spaces, allowing us to make progress towards the rank-rigidity conjecture for CAT(0) spaces.

This is joint work with H. Petyt and A. Zalloum.

Ferrán Valdez. *Affine transformations and isometries of infinite type flat surfaces*

We consider, for a fixed (infinite-type) surface S, the problem of determining all possible groups of isometries and Veech groups (i.e. linear parts of affine transformations) that can arise from flat metrics on S. Using ideas of Aougab, Patel and Vlamis, and of Morales and Delecroix, we show that there are topological restrictions for isometry groups, and on the other side we show evidence for the lack of obstructions for Veech groups.
This is joint work in progress with Artigiani, Randecker, Sadanand and Weitze-Schmithüsen.