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Stochastic and Integral Geometry I (WS 06/07)

Exercise Sheet 6

Please, hand in your solutions at the end of the **lecture** on **Monday, Dec 11th**!

1. Let ν be a probability measure on a topological group T. Show that left-invariance of ν implies its right-invariance as well as its inversion-invariance.

2. Let G_d be the group of rigid motions on $\mathbb{R}^d$ and $\mathcal{C}$ the family of compact subsets of $\mathbb{R}^d$. Show that the mapping $F : G_d \times \mathcal{C} \to \mathcal{C}, (g, C) \mapsto gC$ is continuous.

3. For a convex body $K \in \mathcal{K}$, the *metric projection* onto K is the mapping $p_K : \mathbb{R}^d \to \mathbb{R}^d$ which assigns to each point $x \in \mathbb{R}^d$ its nearest point in K. Show that p_K is a *contraction*, i.e. show that, for all $x, y \in \mathbb{R}^d$,

$$\|p_K(x) - p_K(y)\| \leq \|x - y\| .$$

4*. Two convex bodies $K, L \in \mathcal{K}$ in $\mathbb{R}^d$ are said to *touch* each other, if and only if $K \cap L \neq \emptyset$ and there is a hyperplane E separating K and L (i.e. K and L lie in the two different closed halfspaces bounded by E). For $K, M \in \mathcal{K}$, define

$$G_d(K, M) := \{g \in G_d : K \text{ and } gM \text{ touch}\} .$$

Show that

(a) $\mu(G_d(K, M)) = 0$ (where μ denotes the invariant measure on G_d introduced in the lecture).

(b) the mapping $G_d \to \mathcal{C}, g \mapsto K \cap gM$ is continuous on $G_d \setminus G_d(K, M)$.