Universität Karlsruhe
Institut für Algebra und Geometrie

Prof. Dr. Wolfgang Weil
Dr. Steffen Winter

Stochastic and Integral Geometry I (WS 06/07)

Exercise Sheet 8

Please, hand in your solutions at the end of the **lecture** on **Monday, Jan 15th**!

1. (a) Let $K \in \mathcal{K}$ and B^d the unit ball in $\mathbb{R}^d$. Prove that

$$\int_{G_d} V_0(K \cap gB^d) d\mu(g) = \lambda_d(K + B^d).$$

   (b) Let X be a stationary Poisson process on $\mathbb{R}^d$ with intensity $\gamma > 0$ and $\mathbb{Q}$ a grain distribution concentrated on $\mathcal{K}_0$. Let Y be the associated particle process as in Exercise 4.3. Show that the intensity γ is then given by

$$\gamma = \lim_{r \to \infty} \frac{1}{\lambda_d(rB^d)} \mathbb{E} \sum_{K \in Y} V_0(K \cap rB^d).$$

2. Let Z be a stationary Boolean model in $\mathbb{R}^d$ with intensity $\gamma > 0$ and grain distribution $\mathbb{Q}$ concentrated on $\mathcal{K}_0$. Choose the unit ball B^d as the structuring element and recall that the B^d-distance of $x \in \mathbb{R}^d$ to a set $A \subset \mathbb{R}^d$ is given by

$$d(x, A) := d_{B^d}(x, A) = \inf \left\{ \varepsilon \geq 0 : (x + \varepsilon B^d) \cap A \neq \emptyset \right\}.$$

   Prove that

$$\mathbb{P}(d(0, Z) \leq r) = 1 - \exp\{-\sum_{j=0}^{d} r^{d-j} \kappa_{d-j} \overline{V}_j(Z)\},$$

   where $\overline{V}_j(Z) := \gamma \int_{\mathcal{K}_0} V_j(K) d\mathbb{Q}(K)$.

3. Let Z be a Boolean model as in Exercise 2 above. Fix $d = 3$ and assume that the grain distribution is concentrated on some convex set $K \in \mathcal{K}_0$, i.e. $\mathbb{Q} = \delta_K$. Suppose that the probabilities $p_r := \mathbb{P}(d(0, Z) \leq r)$ are given for $r = 0, 1, 2, 3$.
   Use the formula established in Exercise 2 to derive estimators for γ and the intrinsic volumes $V_1(K)$, $V_2(K)$ and $V_3(K)$ of K in terms of the given probabilities p_0, p_1, p_2, p_3.