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Stochastic and Integral Geometry I (WS 06/07)

Exercise Sheet 9

Please, hand in your solutions at the end of the **lecture** on **Monday, Jan 22nd**!

1. Show that for arbitrary convex sets K and M in $\mathbb{R}^d$ the following relations hold:

   (a) $\operatorname{relint}(K \times M) = \operatorname{relint} K \times \operatorname{relint} M$

   (b) $\operatorname{relint}(K + M) = \operatorname{relint} K + \operatorname{relint} M$

   Here relint$A$ denotes the *relative interior* of the set A, i.e. the interior of A relative to its affine hull.

2. Let L and M be linear subspaces of $\mathbb{R}^d$. Set $l := \dim L$, $m := \dim M$ and $j := \dim(L \cap M)$. Assume that $l + m \geq d$. In the lecture $[L, M]$ was defined as the volume of the parallelopiped spanned by a base $b_L \cup b_M$ of $\mathbb{R}^d$ given by two orthonormal bases b_L of L and b_M of M which are constructed by extending an orthonormal base of $L \cap M$ to L and M, respectively. Show that

$$[L, M] = \frac{1}{\kappa_l \kappa_{d+j-l}} \int_{\mathbb{R}^d} V_j(B_L \cap (B_M + x)) d\lambda_d(x),$$

   where B_L and B_M are the unit balls in L and M, respectively.

3*. Let L and M be linear subspaces of $\mathbb{R}^d$ such that $\dim L + \dim M < d$. Show that

$$\nu(\{\vartheta \in SO_d : \dim(L \cap \vartheta M) \neq 0\}) = 0,$$

   where ν denotes the normalized invariant measure on SO_d.