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Stochastic and Integral Geometry I (WS 06/07)

Exercise Sheet 11

Please, hand in your solutions at the end of the **lecture** on **Monday, Feb 5th**!

1. Let $\mathcal{L}_q^d$ be the set of q-dimensional linear subspaces of $\mathbb{R}^d$ and ν_q the invariant measure on $\mathcal{L}_q^d$ defined in the lecture. Denote by $A|L$ the image of the set $A \subset \mathbb{R}^d$ under the orthogonal projection onto $L \in \mathcal{L}_q^d$. Prove the following formula for the intrinisic volumes: If $K \subset \mathbb{R}^d$ is a convex body and $q, j \in \{1, \dots, d-1\}$ with $j \leq q$, then

$$\int_{\mathcal{L}_q^d} V_j(K|L) d\nu_q(L) = c_{d,q-j}^{d-j,q} V_j(K).$$

2. Let $\mathcal{E}_q^d$ be the set of q-planes in $\mathbb{R}^d$ and μ_q the invariant measure on $\mathcal{E}_q^d$ defined in the lecture. For a convex body $K_0 \subset \mathbb{R}^2$, let $\mathcal{E}_q^d(K_0) := \left\{E \in \mathcal{E}_q^d : E \cap K_0 \neq \varnothing\right\}$ and $\tilde{E}_1, \tilde{E}_2$ be i.i.d. random variables in $\mathcal{E}_q^d(K_0)$ with distribution

$$\mathbb{P} = \frac{\mu_q|_{\mathcal{E}_q^d(K_0)}}{\mu_q(\mathcal{E}_q^d(K_0))}.$$

For $d = 2$ and $q = 1$, $\tilde{E}_1$ and $\tilde{E}_2$ are two random lines in the plane, which almost surely intersect in a single point. Compute the probability that the intersection of $\tilde{E}_1$ and $\tilde{E}_2$ lies in a given convex body $K \subset K_0$. Give the explicit value of this probability, if K_0 is the unit ball and K a ball of radius $\frac{1}{2}$.

3. A needle (of length l) is thrown onto a floor, which is decorated with a pattern of parallel equidistant lines (of distance $d > l$). Determine the probability that the needle hits one of these lines. This problem is known as *Buffon's needle problem.*