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Stochastic and Integral Geometry I (WS 06/07)

Exercise Sheet 12

Please, hand in your solutions at the end of the **lecture** on **Monday, Feb 12th**!

1. Prove the following relation, which is known as the *rotation-sum formula*: For convex bodies $K, M \in \mathcal{K}$ and $j \in \{0, \ldots, d\}$,

$$\int_{SO_d} V_j(K + \vartheta M) d\nu(\vartheta) = \sum_{k=0}^{j} c_{d,d-j}^{d-k,d+k-j} V_k(K) V_{j-k}(M),$$

where the constants $c_{d,d-j}^{d-k,d+k-j}$ are defined as in Exercise 10.2.

2. Show that the curvature measures for sets in the convex ring are *locally defined*, i.e. if $A \subset \mathbb{R}^d$ is a nonempty open set and $K, M \in \mathcal{R}$ such that $K \cap A = M \cap A$, then

$$\Phi_j(K, B) = \Phi_j(M, B)$$

for each Borel subset B of A and $j \in \{0, \ldots, d\}$.

3. Let Z be a stationary Boolean model in $\mathbb{R}^d$ with intensity $\gamma > 0$ and grain distribution $\mathbb{Q}$ concentrated on $\mathcal{K}_0$. Let Y be the underlying particle process. For $j \in \{0, \ldots, d\}$ let $\overline{V}_j(Y) := \gamma \int_{\mathcal{K}_0} V_j(K) d\mathbb{Q}(K)$.

   (a) Prove that

$$\overline{V}_j(Y) = \lim_{r \to \infty} \frac{\mathbb{E} \sum_{K \in Y} V_j(K \cap rB)}{V_d(rB)},$$

   where B is the unit ball in $\mathbb{R}^d$.

   (b) Show that, for $K_0 \in \mathcal{K}$,

$$\mathbb{E} V_d(Z \cap K_0) = (1 - e^{-\overline{V}_d(Y)}) V_d(K_0)$$

   and

$$\mathbb{E} V_{d-1}(Z \cap K_0) = V_{d-1}(K_0)(1 - e^{-\overline{V}_d(Y)}) + V_d(K_0) \overline{V}_{d-1}(Y) e^{-\overline{V}_d(Y)}.$$

   Note that isotropy of the Boolean model is <u>not</u> assumed here.