Universität Karlsruhe (TH)
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Stochastic and Integral Geometry II

Exercise Sheet 2

Please hand in your solutions at the end of the lecture on **Tuesday, May 8th**.

Exercise 1

Show that

$$\begin{aligned} \Phi : \mathbb{R}^d \times \mathcal{C}_0 &\rightarrow \mathcal{C}' \\ (x, C) &\mapsto C + x \end{aligned}$$

is a homeomorphism.

Exercise 2

Let X be a particle process and $X_0 = \{c(C) : C \in X\}$.

a) Is X_0 a point process in $\mathbb{R}^d$?

b) Show that if X is a stationary Poisson process in $\mathcal{C}'$, then X_0 is a (stationary) Poisson process in $\mathbb{R}^d$.

Exercise 3

Let X be a Poisson process in $\mathcal{C}'$ and $\{Y_1, Y_2, \dots\}$ be a measurable enumeration of X as in *Theorem 1.2.6*. Furthermore let $X_0 = \{c(Y_1), c(Y_2), \dots\}$ be the process of center points (as in Exercise 2) and $Z_1 = Y_1 - c(Y_1)$, $Z_2 = Y_2 - c(Y_2), \dots$ the sequence of shapes.

a) Show by an example, that X_0 and $(Z_1, Z_2, \dots)$ need not be independent.

b) Now let X be a stationary Poisson process and $X_0, Z_1, Z_2, \dots$ as above. Show that then $X_0, Z_1, Z_2, \dots$ are independent.