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Stochastic and Integral Geometry II

Exercise Sheet 3

Please hand in your solutions at the end of the lecture on **Tuesday, May 15th**.

Exercise 1

Let $K \in \mathcal{K}$ with $0 \in \operatorname{int} K$. For $C \in \mathcal{C}'$, let $\rho(C)$ be defined as

$$\rho(C) := \inf\{\epsilon > 0 : C \subset \epsilon K\}.$$

Show that $\rho : \mathcal{C}' \to [0, \infty)$ is continuous.

Exercise 2

Let X be a stationary particle process with intensity measure $\Theta \not\equiv 0$. Let $\varphi : \mathcal{C}' \to [0, \infty)$ be measurable, bounded and invariant under translations. In generalization of *Theorem 4.2.3*, show that, for $K \in \mathcal{K}$, $0 \in \operatorname{int} K$ and

$$\bar{\varphi}(X) := \gamma \int_{\mathcal{C}_0} \varphi(C) \, d\mathbb{Q}(C),$$

we have

a)

$$\bar{\varphi}(X) = \frac{1}{V(K)} \mathbb{E} \sum_{C \in X,\, c(C) \in K} \varphi(C),$$

b)

$$\bar{\varphi}(X) = \lim_{r \to \infty} \frac{1}{V(rK)} \mathbb{E} \sum_{C \in X,\, C \subset rK} \varphi(C),$$

c)

$$\bar{\varphi}(X) = \lim_{r \to \infty} \frac{1}{V(rK)} \mathbb{E} \sum_{C \in X,\, C \cap rK \neq \emptyset} \varphi(C).$$

Exercise 3

Let $P \in \mathcal{P}$ be a convex polytope in $\mathbb{R}^2$ and X a stationary, isotropic Poisson process with intensity γ and grain distribution $\mathbb{Q} = \eta(\delta_P)$, where η is a random rotation, uniformly distributed in SO_2, i.e. X has intensity measure

$$\Theta(A) = \gamma \int_{SO_2} \int_{\mathbb{R}^2} 1_A(\vartheta P + x) d\lambda_2(x) d\nu(\vartheta), \quad A \in \mathcal{B}.$$

We consider the (stationary) point process of the intersection points of the boundaries of the particles, which has intensity

$$\gamma_2 = \frac{1}{2\pi} \mathbb{E} \sum_{P_1, P_2 \in X^2_{\neq}} \operatorname{card}(B^2 \cap \operatorname{bd} P_1 \cap \operatorname{bd} P_2).$$

We have $\operatorname{card}(B^2 \cap \operatorname{bd} P_1 \cap \operatorname{bd} P_2) \in \mathbb{N}_0$ a.s.

Show that $\gamma_2 = \frac{\gamma^2}{\pi} V_1(\operatorname{bd}(P))^2$.

Hint: Use the Principal Kinematic Formula for sets in the convex ring in $\mathbb{R}^2$!