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Stochastic and Integral Geometry II

Exercise Sheet 4

Please hand in your solutions at the end of the lecture on **Tuesday, May 22nd**.

Exercise 1

Let X be a stationary and isotropic process of convex particles in $\mathbb{R}^d$ and $L \subset \mathbb{R}^d$ a q-dimensional subspace. Show that

$$X \cap L = \{K \cap L : \, K \in X, \, K \cap L \neq \emptyset\}$$

is a stationary and isotropic process of convex particles in L and that

$$\overline{V}_j(X \cap L) = \alpha_{djq} \overline{V}_{d+j-q}(X), \quad \text{for } 0 \leq j \leq q.$$

Exercise 2

Let X be a stationary Poisson particle process in $\mathbb{R}^2$ of strictly convex particles with intensity γ and grain distribution $\mathbb{Q}$. Strict convexity means, that the boundaries of the particles do not contain any lines. We now assign to any particle $K \in X$ a point $x(K) \in K$, the so called lower tangent point, which is the point in K with smallest y-coordinate. Note that $x(K)$ is unique by the strict convexity of K.

a) Show that the mapping $K \mapsto x(K)$ is continuous.

b) Show that

$$\tilde{X} := \{x(K) : \, K \in X\}$$

is a stationary Poisson process with intensity γ.

c) Now consider

$$X' := \left\{ y \in \tilde{X} : y \notin \bigcup_{K \in X, \, x(K) \neq y} K \right\}.$$

Show that X' is a stationary point process with intensity

$$\gamma' = \gamma e^{-\overline{V_2}(X)}.$$

Is X' a Poisson process?

Exercise 3

Let $A \subset \mathbb{R}^d$ be nonempty and locally finite with $\operatorname{conv} A = \mathbb{R}^d$. For $x \in A$, let

$$C(x, A) := \{y \in \mathbb{R}^d : ||x - y|| \leq \inf_{z \in A} ||z - y||\}$$

be the *Voronoi cell* of x (with respect to A). Show that $C(x, A) \in \mathcal{K}'$. Show by an example that the condition $\operatorname{conv} A = \mathbb{R}^d$ is not necessary for the conclusion $C(x, A) \in \mathcal{K}'$ (for all $x \in A$).