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Stochastic and Integral Geometry II

Exercise Sheet 6

Please hand in your solutions at the end of the lecture on **Tuesday, June 5th**.

Exercise 1

Let X be a random mosaic and $\mathcal{X}^{(j,k)}$ the process of (j,k)-face stars of X. Show that $\mathcal{X}^{(j,k)}$ can be represented in a one-to-one manner as a point process Y on the convex ring $\mathcal{R}' = \mathcal{R} \setminus \{\emptyset\}$.

Exercise 2

Let X be a stationary random mosaic, $\gamma^{(0)}$ the intensity of vertices, $d_1^{(1)}$ the specific length of the edge process $X^{(1)}$, and $l_{01} := v_1^{(0,1)}$ the mean length of the typical edge star ($(0,1)$-face star) of X. Show that

$$\gamma^{(0)} l_{01} = 2 d_1^{(1)}.$$

Exercise 3 *(Variant of Exercise 5.3)*

Let M be a locally compact space with countable base and X a point process on $\mathcal{C}' \times M$ which is stationary in the sense, that $\mathbb{P}_X$ is invariant under all mappings

$$T_x : (C,m) \mapsto (C+x, m), \quad x \in \mathbb{R}^d.$$

Assume that the intensity measure Θ of X fulfills $\Theta(\mathcal{A} \times M) < \infty$ for all compact $\mathcal{A} \subset \mathcal{C}'$, and $\Theta \not\equiv 0$.

a) Show that there exists a constant $\gamma \in (0,\infty)$ and a probability measure $\mathbb{Q}$ on $\mathcal{C}_0 \times M$ such that

$$\Theta = \Phi(\gamma \lambda_d \otimes \mathbb{Q}),$$

where

$$\Phi : (x, (C,m)) \mapsto (C+x, m).$$

b) Show that for measurable $f : \mathcal{C}' \times M \to [0,\infty)$

$$\mathbb{E} \sum_{(C,m)\in X} f(C,m) = \gamma \int_{\mathcal{C}_0 \times M} \int_{\mathbb{R}^d} f(C+x, m) d\lambda_d(x) d\mathbb{Q}(C,m).$$