Universität Karlsruhe (TH)
Institut für Algebra und Geometrie

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Stochastic and Integral Geometry II

Exercise Sheet 7

Please hand in your solutions at the end of the lecture on **Tuesday, June 12th**.

Exercise 1

Let X be a stationary random mosaic and $x \in \mathbb{R}^d$. Show that a.s. there is a unique cell Z_x of X with $x \in \operatorname{int} Z_x$. Z_x is called the *x-cell* of X. Z_x is a random polytope. Let $f : \mathcal{K}' \to [0, \infty)$ be translation invariant and measurable. Show that

$$\mathbb{E} f(Z_0) = \mathbb{E} \int_{[0,1]^d} f(Z_x) d\lambda_d(x).$$

Exercise 2

Let $\tilde{X} \subset \mathbb{R}^2$ be a stationary Poisson process with intensity $\tilde{\gamma}$ and X the Voronoi mosaic generated by $\tilde{X}$. X is a stationary random mosaic. Show that the specific edge length $d_1^{(1)}$ of X satisfies

$$d_1^{(1)} = 2\sqrt{\tilde{\gamma}}.$$