Universität Karlsruhe (TH)
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Stochastic and Integral Geometry II

Exercise Sheet 8

Please hand in your solutions at the end of the lecture on **Tuesday, June 19th**.

Exercise 1

a) Let $d = 2$ and $k \in \{3, 4, 5, 6\}$. Construct stationary random mosaics X and Y in $\mathbb{R}^2$, such that each vertex of X lies in exactly k edges and each cell of Y has exactly k edges.

Show that such mosaics are not possible for $k \geq 7$.

Show also that a stationary random mosaic X in $\mathbb{R}^2$, such that each vertex of X lies in exactly k edges and each cell has exactly k edges, only exists for $k = 4$.

b) Let $\alpha \in [3, 6]$. Show that there are stationary random mosaics X in $\mathbb{R}^2$ with $n_{01} = \alpha$ and Y in $\mathbb{R}^2$ with $n_{20} = \alpha$.

Exercise 2

A mosaic m in $\mathbb{R}^d$ is *simplicial*, if each cell of m is a simplex (a simplex is the convex hull of affinely independent points). In a simplicial mosaic, all faces are simplices. Let X be a stationary random mosaic in $\mathbb{R}^d$, which is a.s. simplicial. Show that

$$(1 - (-1)^{d-k})\gamma^{(k)} = \sum_{j=k+1}^{d} (-1)^{d-j} \binom{j+1}{k+1} \gamma^{(j)}$$

holds, for $k = 0, ..., d - 1$.
Interpret the result for $k = d - 1$.