Exercise 1 (Completeness of submanifolds).
Let $(M, g)$ be a complete Riemannian manifold and let $N \subset M$ be a closed embedded submanifold. Show that $N$ with the induced metric is again complete.

Exercise 2.
Let $(M, g)$ be a Riemannian manifold and let further $f: M \to \mathbb{R}$ be a smooth function on $M$ with the property $|\text{grad } f| \equiv 1$. Show that integral curves of $\text{grad } f$ are geodesics.

Exercise* 3 (Riemannian coverings).
Let $p: \tilde{M} \to M$ be a smooth covering of a Riemannian manifold $(M, g)$. Show that $\tilde{M}$ admits a metric $\tilde{g}$ such that $p$ becomes a local isometry. Show that $(\tilde{M}, \tilde{g})$ is complete if and only if $(M, g)$ is.

Exercise 4 (Hopf-Rinow Theorem).
Give an example of a non-complete connected Riemannian manifold $M$ such that any two points $p$ and $q$ can be joined by a distance realising geodesic in $M$.

Due: Wednesday May 3rd, 2017, before the exercise class.