Karlsruher Institut für Technologie

Institut für Algebra und Geometrie

Dr. F. Galaz-García

Comparison Geometry in Summer 2017

Exercise sheet 2

Exercise 1 (Hadamard-Cartan)**.**
Prove or disprove: Any complete Riemannian metric on $S^1 \times S^2$ must have positive sectional curvature somewhere.

Exercise 2 (Nonpositive curvature I)**.**
Let (M, g) be a simply-connected complete Riemannian manifold with nonpositive sectional curvature. Show that every isometry $\Phi\colon M \to M$ of finite order in $\mathrm{Isom}(M)$, i.e. there exists $n \in \mathbb{N}$ such that $\Phi^n = \mathrm{id}_M$ has a fixed point.

Exercise 3 (Nonpositive curvature II)**.**
Let (M, g) be a complete Riemannian manifold with nonpositive sectional curvature. Show that $\pi_1(M)$ is torsion free, i.e. that there are no finite order elements contained in the fundamental group.

Due: Wednesday May 10, 2017, before the exercise class.