Karlsruher Institut für Technologie

Institut für Algebra und Geometrie

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Comparison Geometry in Summer 2017

Exercise sheet 3

Exercise 1.
Let $N_1, N_2 \subset M$ be closed, disjoint submanifolds of a compact Riemannian manifold (M, g). Show that there is a minimizing geodesic $\gamma\colon [0,1] \to M$ between N_1 and N_2 which is perpendicular to both submanifolds, i.e. $\dot\gamma(0) \perp \mathrm{T}_{\gamma(0)} N_1$ and $\dot\gamma(1) \perp \mathrm{T}_{\gamma(1)} N_2$.

Exercise 2.
Let $f\colon (-\varepsilon, \varepsilon) \times [0,a] \to M$ be a variation of a piecewise smooth curve $c\colon [0,a] \to M$ in a Riemannian manifold M. Prove that on any rectangle $(-\varepsilon, \varepsilon) \times [t_i, t_{i+1}]$, where f is smooth, one has

$$\frac{\mathrm{D}}{\mathrm{d}\,s}\frac{\partial f}{\partial t} = \frac{\mathrm{D}}{\mathrm{d}\,t}\frac{\mathrm{d}\,f}{\mathrm{d}\,s}.$$

Exercise 3.
Let M be a Riemannian manifold and let

$$\Omega_{p,q} := \{c\colon [0,1] \to M \mid c \text{ is piecewise smooth and } c(0) = p, c(1) = q\}.$$

Show that a constant speed curve $c \in \Omega_{p,q}$ minimizes the arc length functional $L\colon \Omega_{p,q} \to [0,\infty)$ if and only if it minimizes the energy functional $E\colon \Omega_{p,q} \to [0,\infty)$.

Due: Wednesday May 17, 2017, before the exercise class.