Karlsruher Institut für Technologie

Institut für Algebra und Geometrie

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Comparison Geometry in Summer 2017

Exercise sheet 4

Exercise 1.
Let (M, g) be a complete simply-connected nonpositively curved Riemannian manifold and let $\lambda\colon \mathbb{R} \to M$ be a geodesic parametrised by arc length. For a point $p \in M \setminus \mathrm{Im}(\lambda)$ define $\mathrm{d}(s) = \mathrm{d}_g(p, \lambda(s))$.

(a) Consider a family of geodesics $\gamma_s\colon [0, \mathrm{d}(s)] \to M$ from p to $\lambda(s)$ and show that

$$\frac{1}{2}E'(s) = \big\langle \lambda'(s), \gamma_s'(\mathrm{d}(s)) \big\rangle.$$

(b) Conclude that s_0 is a critical point of d if and only if $\big\langle \lambda'(s_0), \gamma_{s_0}'(d(s_0)) \big\rangle = 0$.

Exercise 2.
Let (M, g) be a simply-connected complete Riemannian manifold of nonpositive curvature and let $\lambda\colon \mathbb{R} \to M$ be a geodesic parametrized by arc lenght. For a point $p \in M \setminus \mathrm{Im}(\lambda)$ define $\mathrm{d}(s) = \mathrm{d}_g(p, \lambda(s))$. Show that

$$\frac{1}{2}E''(s) > 0$$

and conclude that d has exactly one critical point, which is a minimum.

Exercise 3.
Show that completeness is necessary in the theorem of Bonnet-Myers to conclude that the fundamental group is finite.

Due: Wednesday May 24, 2017, before the exercise class.