Exercise 1.
Let \((M, g)\) be a complete simply-connected nonpositively curved Riemannian manifold and let \(\lambda : \mathbb{R} \to M\) be a geodesic parametrised by arc length. For a point \(p \in M \setminus \text{Im}(\lambda)\) define \(d(s) = d_g(p, \lambda(s))\).

(a) Consider a family of geodesics \(\gamma_s : [0, d(s)] \to M\) from \(p\) to \(\lambda(s)\) and show that
\[
\frac{1}{2} E''(s) = \langle \lambda'(s), \gamma'_s(d(s)) \rangle.
\]

(b) Conclude that \(s_0\) is a critical point of \(d\) if and only if \(\langle \lambda'(s_0), \gamma'_{s_0}(d(s_0)) \rangle = 0\).

Exercise 2.
Let \((M, g)\) be a simply-connected complete Riemannian manifold of nonpositive curvature and let \(\lambda : \mathbb{R} \to M\) be a geodesic parametrized by arc length. For a point \(p \in M \setminus \text{Im}(\lambda)\) define \(d(s) = d_g(p, \lambda(s))\). Show that
\[
\frac{1}{2} E''(s) > 0
\]
and conclude that \(d\) has exactly one critical point, which is a minimum.

Exercise 3.
Show that completeness is necessary in the theorem of Bonnet-Myers to conclude that the fundamental group is finite.

Due: Wednesday May 24, 2017, before the exercise class.