

Comparison Geometry in Summer 2017 Exercise sheet 5

Exercise 1.

Let M^n be a compact manifold with positive sectional curvature.

- (i) Show that if n is even, orientability is necessary to conclude, via Synge's Theorem, that M is simply connected.
- (ii) Show that if n is odd, one cannot conclude that M is simply connected.

Exercise 2.

Show that $\mathbb{R}P^2 \times \mathbb{R}P^2$ has a metric with positive Ricci curvature but does not admit a metric with positive sectional curvature.

Exercise 3.

Consider the gradient vector field $V = \nabla \text{dist}(0, \cdot)$ on \mathbb{R}^n and let $\gamma(t) = t \frac{\partial}{\partial x_n}$.

- (i) Prove that $V = \sum_{i=1}^n \frac{x_i}{r} \frac{\partial}{\partial x_i}$ for $r^2 = \sum_{i=1}^n x_i^2$.
- (ii) Let $A(t) = \nabla V|_{\gamma(t)}$. Prove that

$$A(t) = \frac{1}{t} I$$

with respect to the basis $\left\{ \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_{n-1}} \right\}$.