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Comparison Geometry in Summer 2017
Exercise sheet 9

Exercise 1.
Show that if equality holds in the Bishop-Gromov inequality for some $0 < r < R \leq \mathrm{diam}(M)$, then $B_R(p)$ is isometric to a ball of radius R in the simply-connected space of constant sectional curvature k.

Exercise 2.
Let M be a complete non-compact Riemannian manifold. A *ray* in M is a geodesic $\gamma\colon [0,\infty) \to M$ such that $d(\gamma(0),\gamma(t)) = t$ for all t. Assume that M has non-negative sectional curvature and let $\gamma, \sigma\colon [0,\infty) \to M$ be geodesics such that $\gamma(0) = \sigma(0)$. Show that, if γ is a ray and $\angle(\gamma'(0), \sigma'(0)) < \pi/2$, then
$$\lim_{t\to\infty} d(\sigma(0),\sigma(t)) = \infty.$$

Due: Wednesday July 5, 2017, before the exercise class.