Exercise 1.
Show that if equality holds in the Bishop-Gromov inequality for some $0 < r < R \leq \text{diam}(M)$, then $B_R(p)$ is isometric to a ball of radius $R$ in the simply-connected space of constant sectional curvature $k$.

Exercise 2.
Let $M$ be a complete non-compact Riemannian manifold. A ray in $M$ is a geodesic $\gamma: [0, \infty) \rightarrow M$ such that $d(\gamma(0), \gamma(t)) = t$ for all $t$. Assume that $M$ has non-negative sectional curvature and let $\gamma, \sigma: [0, \infty) \rightarrow M$ be geodesics such that $\gamma(0) = \sigma(0)$. Show that, if $\gamma$ is a ray and $\angle(\gamma'(0), \sigma'(0)) < \pi/2$, then
\[
\lim_{t \rightarrow \infty} d(\sigma(0), \sigma(t)) = \infty.
\]

Due: Wednesday July 5, 2017, before the exercise class.