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Comparison Geometry in Summer 2017

Exercise sheet 11

Exercise 1.
Prove the following theorem.

Theorem (Gromov, 1980). For a Riemannian manifold M and $\tilde{x} \in \tilde{M}$, one can always find generators $\{\gamma_1, \ldots, \gamma_m\}$ for the fundamental group $\Gamma = \pi_1(M)$ such that $d(\tilde{x}, \gamma_i(\tilde{x})) \leq 2\mathrm{diam}(M)$ and such that all relations for Γ in these generators are of the form $\gamma_i \cdot \gamma_j \cdot \gamma_k^{-1} = 1$.

Exercise 2.
Use the preceding exercise and the Bishop-Gromov inequality to prove the following theorem.

Theorem (Anderson, 1990). For $n \in \mathbb{N}$, $k \in \mathbb{R}$, and $v, D \in (0, \infty)$, let $\mathfrak{M}(n, k, v, D)$ denote the class of compact Riemannian n-manifolds with $\mathrm{Ric} \geq (n-1)k$, $\mathrm{vol} \geq v$, and $\mathrm{diam} \leq D$. Then, for fixed n, k, v, D, there are only finitely many fundamental groups among the manifolds in $\mathfrak{M}(n, k, v, D)$.

Due: Wednesday July 19, 2017, before the exercise class.