Karlsruher Institut für Technologie

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Comparison Geometry in Summer 2015

Exercise sheet 1

Exercise 1 (Completeness of submanifolds)**.**
Let (M, g) be a complete Riemannian manifold and let $N \subset M$ be a closed embedded submanifold. Show that N with the induced metric is again complete.

Exercise 2.
Let (M, g) be a Riemannian manifold and let further $f \colon M \to \mathbb{R}$ be a smooth function on M with the property $|\operatorname{grad} f| \equiv 1$. Show that integral curves of $\operatorname{grad} f$ are geodesics.

Exercise* 3 (Riemannian coverings)**.**
Let $p \colon \tilde{M} \to M$ be a smooth covering of a Riemannian manifold (M, g). Show that $\tilde{M}$ admits a metric $\tilde{g}$ such that p becomes a local isometry. Show that $(\tilde{M}, \tilde{g})$ is complete if and only if (M, g) is.

Exercise 4 (Spaces of constant curvature)**.**
Show that $\mathbb{R}^n, \mathbb{H}^n$ and S^n with their standard metrics are complete Riemannian manifolds.

Exercise 5 (Hopf-Rinow Theorem)**.**
Give an example of a non-complete connected Riemannian manifold M such that any two points p and q can be joined by a distance realising geodesic in M.

Due: Wednesday April 22, 2015, before the exercise class.