Comparison Geometry in Summer 2015
Exercise sheet 5

Let $(M,g)$ be a Riemannian manifold and let $\gamma : [0,a] \to M$ be a geodesic with a variation $f : (-\varepsilon, \varepsilon) \times [0,a] \to M$. Let $V$ be the variational field of $f$ and let $E$ be the energy function of the variation. Let

$$I_a(V,V) := \int_0^a \{ \langle V', V' \rangle - \langle R(\gamma', V)\gamma', V \rangle \} dt.$$

**Exercise 1.**
Show that if $f$ is a proper variation, then $\frac{1}{2}E''(0) = I_a(V,V)$.

**Exercise 2.**
Show that if $f$ is a general variation, then

$$\frac{1}{2}E''(0) = I_a(V,V) - \langle \frac{D}{ds} \frac{\partial f}{\partial s}, \gamma' \rangle(0,0) + \langle \frac{D}{ds} \frac{\partial f}{\partial s}, \gamma' \rangle(0,a).$$

Due: Wednesday May 20th, 2015, before the exercise class.