Karlsruher Institut für Technologie

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Comparison Geometry in Summer 2015

Exercise sheet 6

Exercise* 1 (Shorter curves in positive curvature)**.**
Let (M^{2n}, g) be an orientable and positively curved Riemannian manifold. Let γ be a closed geodesic in M, i.e. $\gamma\colon S^1 \to M$ is an immersion that is a geodesic at all of its points. Show that γ is homotopic to some closed curve c in M with $L(c) < L(\gamma)$.

Exercise 2.
Show that the hypothesis $\sec \geq \delta > 0$ in the corollary to the theorem of Bonnet-Myers is necessary. In other words, find a complete, non-compact Riemannian manifold whose sectional curvature is positive and not bounded away from 0.

Exercise 3.
Let M^n be a compact manifold with positive sectional curvature.

(i) Show that if n is even, orientability is necessary to conclude, via Synge's Theorem, that M is simply connected.

(ii) Show that if n is odd, one cannot conclude that M is simply connected.

Exercise 4.
Show that completeness is necessary in the theorem of Bonnet-Myers to conclude that the fundamental group is finite.

Exercise 5.
Show that $\mathbb{R}P^2 \times \mathbb{R}P^2$ has a metric with positive Ricci curvature but does not admit a metric with positive sectional curvature.

Due: Wednesday May 27th, 2015, before the exercise class.