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Global Differential Geometry
Exercise sheet 10

The real *intersection form* Q_M of an oriented smooth compact 4-manifold is defined as

$$Q_M : \mathrm{H}^2_{\mathrm{dR}}(M) \times \mathrm{H}^2_{\mathrm{dR}}(M) \to \mathbb{R}, ([\omega], [\eta]) \mapsto \int_M \omega \wedge \eta,$$

where $\mathrm{H}^*_{dR}(M)$ denotes de Rham cohomology and $[\omega]$ is the cohomology class of the closed differential form ω.

Exercise 1

Show that Q_M is a well-defined symmetric bilinear form.

Exercise 2

Compute the intersection form of $\mathrm{S}^2 \times \mathrm{S}^2$.

Hint: $\mathrm{H}^2_{\mathrm{dR}}(\mathrm{S}^2 \times \mathrm{S}^2) \cong \mathbb{R}^2$.

Exercise 3

Let M be a compact simply connected 4-manifold that has an orientation reversing diffeomorphism. Compute the signature of Q_M.