The real \textit{intersection form} $Q_M$ of an oriented smooth compact 4-manifold is defined as

\[ Q_M : H^2_{dR}(M) \times H^2_{dR}(M) \to \mathbb{R}, ([\omega],[\eta]) \mapsto \int_M \omega \wedge \eta, \]

where $H^*_{dR}(M)$ denotes de Rham cohomology and $[\omega]$ is the cohomology class of the closed differential form $\omega$.

\textbf{Exercise 1}

Show that $Q_M$ is a well-defined symmetric bilinear form.

\textbf{Exercise 2}

Compute the intersection form of $S^2 \times S^2$.

\textbf{Hint:} $H^2_{dR}(S^2 \times S^2) \cong \mathbb{R}^2$.

\textbf{Exercise 3}

Let $M$ be a compact simply connected 4-manifold that has an orientation reversing diffeomorphism. Compute the signature of $Q_M$. 