

Global Differential Geometry

Exercise sheet 3

Exercise 1 (Cheng's maximal diameter sphere theorem)

Let M be a n -dimensional complete Riemannian manifold with $\text{Ric} \geq (n - 1)$. Show that $\text{diam}M \leq \pi$ and equality holds if and only if M is isometric to S^n .

Exercise 2

Do the following spaces admit an intrinsic metric inducing their topology?

- The rational numbers;
- The union of graph of the function $\sin(1/x)$, $x > 0$ and the y -axis;
- $X = \bigcup_{k=1}^{\infty} L_k \cup L_{\infty} \subset \mathbb{R}^2$ consisting of the lines $L_k = [(0, 0), (\cos(1/k), \sin(1/k))]$ and $L_{\infty} = [(0, 0), (1, 0)]$.

Exercise 3

For a metric space (X, d) , let

$$\hat{d}(x, y) = \inf\{L(\gamma) \mid \gamma \text{ is a curve connecting } x \text{ and } y\}$$

for all $x, y \in X$.

Show that $\hat{\hat{d}} = \hat{d}$, so (X, \hat{d}) is a length space.