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Global Differential Geometry

Exercise sheet 4

Exercise 1

Let $2 \leq p \in \mathbb{N}$ and let $1 \leq q_1, \ldots, q_k < p$ be natural numbers relatively prime to p. Show that the group of p-th roots of unity $E_p = \{z \in \mathbb{C} \mid z^p = 1\}$ acts freely and properly discontinuously on

$$\mathbb{S}^{2k-1} = \{(z_1, \ldots, z_k) \in \mathbb{C}^k \mid \sum_{i=1}^{k} |z_i|^2 = 1\}$$

via

$$z \cdot (z_1, \ldots, z_k) = (z^{q_1} z_1, \ldots, z^{q_k} z_k).$$

Exercise 2

Show that the n-dimensional sphere $\mathbb{S}^n$ is diffeomorphic to $\mathrm{SO}(n+1)/\mathrm{SO}(n)$.

Exercise 3

(a) Give an example of a Lie group whose exponential map is not surjective.

(b) Give an example of a Lie group whose exponential map is not injective.

Exercise 4

Show that a subgroup H of a Lie group G is discrete if and only if there is a neighborhood of $e \in G$ that contains no other elements of H.

Due: Friday, 9.23.2014, during the exercise class.