Karlsruher Institut für Technologie

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Global Differential Geometry

Exercise sheet 5

Exercise 1

Let M be a smooth manifold with a smooth action of a Lie group G. Show that for every $x \in M$ the isotropy group G_x is a closed subgroup of G.

Exercise 2

Let G and M be as in Exercise 1 and suppose that the G-action is transitive. Prove that for all $x, y \in M$ the isotropy groups G_x and G_y are conjugate subgroups of G.

Exercise 3

Let $\langle \cdot, \cdot \rangle$ denote the usual inner product in $\mathbb{R}^n$.

(a) Show that the real Stiefel manifold, defined by

$$V_k(\mathbb{R}^n) := \{(v_1, \ldots, v_k) \mid v_i \in \mathbb{R}^n,\ \langle v_i, v_j \rangle = \delta_{ij},\ i, j = 1 \ldots, k\},$$

is a homogeneous space.

(b) Show that

$$V_k(\mathbb{R}^n) = \mathrm{O}(n)/\mathrm{O}(n-k).$$

for an appropriate realization of $\mathrm{O}(n-k)$ as a subgroup of $\mathrm{O}(n)$

Exercise 4

(a) Prove that the Grassmann manifold

$$G_k(\mathbb{R}^n) := \{E \subset \mathbb{R}^n \mid E \text{ is a } k\text{-dimensional subspace of } \mathbb{R}^n\}$$

is a homogeneous space.

(b) Show that

$$G_k(\mathbb{R}) = \mathrm{O}(n)/(\mathrm{O}(k) \times \mathrm{O}(n-k)).$$

Due: Friday, 30.5.2014, during the exercise class.