

Global Differential Geometry

Exercise sheet 8

Exercise 1

Show that the Klein bottle is a fiber bundle with fiber \mathbb{S}^1 and structure group \mathbb{Z}_2 acting by reflection on $\mathbb{S}^1 \subset \mathbb{R}^2$.

Exercise 2

Let $P \rightarrow M$ be a principal bundle with group G . If H is a Lie subgroup of G , then the quotient P/H is an H -principal bundle. Show that $P/H \rightarrow M$ admits a global section if and only if the structure group of $P \rightarrow M$ is reducible to H .

Exercise 3

Let M be a compact Riemannian manifold with an isometric effective action of a compact Lie group G . Suppose that $M/G \cong [-1, +1]$, the isotropy group of the orbits in $(-1, +1)$ is $H \leq G$ and the isotropy group of the orbits at the endpoints ± 1 is $K_{\pm} \leq H$, respectively. Use the Slice Theorem to obtain a description of M as the union of disk bundles over the orbits G/K_{\pm} .