Karlsruher Institut für Technologie

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Global Differential Geometry
Exercise sheet 9

Exercise 1

Let M be a smooth manifold and let $\mathcal{X}(M)$ be the set of smooth vector fields on M. Show that the Lie derivative $\mathcal{L} : \mathcal{X}(M) \times \mathcal{X}(M) \to \mathcal{X}(M)$ is not a connection.

Exercise 2

Let M be a smooth manifold with a linear connection ∇. Show that the map

$$\begin{aligned} \mathcal{X}(M) \times \mathcal{X}(M) \times \mathcal{X}(M) &\to \mathcal{X}(M) \\ (X, Y, Z) &\mapsto \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z \end{aligned}$$

defines a $(3, 1)$ tensor field.

Exercise 3

A Riemannian manifold M is *isotropic* at $p \in M$ if there exists a Lie group G acting by isometries on M such that the isotropy group $G_p \leq G$ acts transitively on the unit tangent sphere at p. We say that M is *isotropic* if it is isotropic at every point. Suppose that M is a complete Riemannian manifold that is isotropic. Show that M is homogeneous.
Hint: Given $p, q \in M$, consider the midpoint of a geodesic in M.

Exercise 4

Show that a homogeneous Riemannian manifold is complete.

Due: Friday,4.7.2014, during the exercise class.