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Global Differential Geometry

Exercise sheet 10

Exercise 1

Let (M, g) be a Riemannian manifold. If f is a smooth function on M such that $\text{grad} f = 1$, show that the integral curves of $\text{grad} f$ are geodesics.

Exercise 2

In Euclidean space, the parallel transport of a vector between two points does not depend on the curve joining the two points. Show, by example, that this fact may not be true on an arbitrary Riemannian manifold.

Exercise 3

Show that a closed embedded submanifold of a complete Riemannian manifold is complete in the induced Riemannian metric.

Exercise 4

Let G be a Lie group with a bi-invariant metric g. Show that the Riemannian curvature tensor of g can be computed as follows:

$$R(X, Y)Z = \frac{1}{4}[Z, [X, Y]]$$

whenever X, Y and Z are left-invariant vector fields on G.

Due: Friday, 11.7.2014, during the exercise class.