Institut für Algebra und Geometrie

Prof. Dr. W. Tuschmann
Dr. F. Galaz-García

Karlsruher Institut für Technologie

Global Differential Geometry

Exercise sheet 11

Exercise 1

Let (M, g) be a Riemannian manifold with constant sectional curvature K. Prove that the curvature tensor of g is given by

$$R(X, Y)Z = K(\langle Y, Z\rangle X - \langle X, Z\rangle Y).$$

Exercise 2

Let (M, g) be a Riemannian manifold with constant sectional curvature K, and let γ be a unit speed geodesic in M. Show that the normal Jacobi fields along γ vanishing at $t = 0$ are the vector fields of the form

$$J(t) = u(t)E(t),$$

where E is any parallel normal vector field along γ, and $u(t)$ is given by

$$u(t) = \begin{cases} t, & \text{if } K = 0; \\ R\sin(t/R), & \text{if } K = 1/R^2 > 0; \\ R\sinh(t/R), & \text{if } K = -1/R^2 < 0. \end{cases}$$

Hint: Use the Jacobi equation and Exercise 1.

Exercise 3

Let G be a Lie group with a bi-invariant metric and let X, Y be left-invariant unit vector fields on G. Show that if X and Y are orthogonal, then the sectional curvature $K(\sigma)$ of the plane σ spanned by X and Y is given by

$$K(\sigma) = \frac{1}{4}||[X, Y]||^2.$$

Hint: Use Exercise 4 in Sheet 10.

Due: Friday, 18.7.2014, during the exercise class.