Abstract of the talk by
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Lower bounds for the Yamabe invariant

In this talk I want to give an overview over some old and new results about the (smooth) Yamabe invariant in joint work with M. Dahl and E. Humbert. We will see for example that the Yamabe invariant of a simply connected compact spin manifold of dimension 5 is between 45 and 79. Similar estimates hold for 2-connected compact spin manifolds with vanishing index.

Let us give some more details. The conformal Yamabe constant of a compact riemannian manifold (M, g_0) is defined as

$$Y(M, [g_0]) := \inf \int_M \mathrm{scal}^g \; dv^g$$

where the infimum runs over all metrics g of volume 1 in $[g_0]$. The smooth Yamabe invariant of M is then defined as

$$\sigma(M) := \sup Y(M, [g_0])$$

where the supremum runs over all conformal classes $[g_0]$ on M.

These invariants are tightly related to the existence of metrics of constant scalar curvature in a given conformal class. The invariant $\sigma(M)$ is positive iff M carries a metric of positive scalar curvature.

We have proven a formula that estimates the behaviour of $\sigma(M)$ under performing surgery at M, namely if N is obtained by surgery of codimension $k \geq 3$ from M, then

$$\sigma(N) \geq \min\{\sigma(M), \Lambda_{n,k}\},$$

where $\Lambda_{n,k} > 0$ only depends on $n = \dim M$ and k.

The constants $\Lambda_{n,k} > 0$ arise as conformal Yamabe constants of certain limit spaces which are products of rescaled spheres with the standard hyperbolic spaces.

In recent work we found an efficient method to control the Yamabe constants of products spaces provided that both factors are of dimension at least 3. This formula yields positive lower bounds for $\Lambda_{n,k}$ in the case $k \notin \{1, n-3\}$. We also found a method to compare the conformal Yamabe constant of our model spaces with the conformal Yamabe invariant of spaces like $\mathbb{R}^2 \times S^2$, $\mathbb{R}^3 \times S^2$ and $\mathbb{R}^2 \times S^3$. The Yamabe constants of the latter spaces were recently calculated by Petean and Ruiz. This method is e.g. sufficient to control $\Lambda_{5,1}$ and $\Lambda_{5,2}$ and thus yields the explicit bound mentioned above for 5-manifolds.