# Differential Geometry in the Large 2022

## Christian Bär (Potsdam)

### Curvature flexibility for $C^{1,1}$-metrics
**Friday, 11:20**

We show a general approximation theorem which implies for instance the following: For any $C^2$-metric $g$ on a smooth manifold $M$ of whatever topology there exists a $C^{1,1}$-metric $g'$ such that:

1 $g'$ is $C^1$-close to $g$

2 $g'$ is smooth and flat on an open and dense subset of $M$.

Obviously, such a metric $g'$ cannot be $C^2$ in general. The proof of the approximation theorem relies on a method suggested by Gromov called "local flexibility". We will present some further applications.

## Christoph Böhm (Münster)

### Non-Compact Einstein Manifolds with Symmetry
**Wednesday, 10:00**

For Einstein manifolds with negative scalar curvature admitting an isometric action of a Lie group $G$ with compact, smooth orbit space, we show the following rigidity result: The nilradical $N$ of $G$ acts polarly, and the $N$-orbits can be extended to minimal Einstein submanifolds. As an application, we prove the Alekseevskii conjecture: Any homogeneous Einstein manifold with negative scalar curvature is diffeomorphic to a Euclidean space. This is joint work with R. Lafuente.

## Nicola Cavallucci (Karlsruhe)

### Collapsing of manifolds with non-positive curvature
**Thursday, 10:00**

It is classical that a sequence of Riemannian manifolds $X_n$ with bounded curvature, bounded diameter and a uniform lower bound on the injectivity radius converge in $C^1$-norm to a Riemannian manifold of the same dimension. It is natural to ask what happens in the collapsing case, that is when the injectivity radius goes to zero along the sequence. The main result of the talk is to show that in this setting each manifold $X_n$ admits a covering of finite universal degree which is fibrated in flat tori and these tori collapse to a point in the limit. In a more geometric way we show that the only way to decrease the dimension is to collapse circles to points. We will discuss the generalization of this theorem to CAT(0)-spaces.
The is based on a joint work with A. Sambusetti.

## Anand Dessai (Fribourg)

### Moduli spaces of metrics of nonnegative sectional curvature
**Tuesday, 11:20**

We review recent results on the topology of moduli spaces of metrics of nonnegative sectional curvature on closed manifolds.

## Bernhard Hanke (Augsburg)

### Lipschitz rigidity for scalar curvature
**Wednesday, 11:20**

Lower scalar curvature bounds on spin Riemannian manifolds exhibit remarkable rigidity properties determined by the index theory of Dirac operators. A fundamental result of Llarull states that there is no smooth Riemannian metric on the $n$-sphere which dominates the round metric and whose scalar curvature is greater than or equal to the scalar curvature of the round metric, except the round metric itself. A similar result holds for smooth comparison maps from spin Riemannian manifolds to round spheres. In a joint work with Simone Cecchini and Thomas Schick, we discuss these results for Riemannian metrics with regularity smaller than $C^1$ and Lipschitz comparison maps. This uses spectral properties of Dirac operators twisted with Lipschitz bundles and the theory of quasi-regular maps.

## Lynn Heller (Hannover)

### Complex analytic methods in constructing harmonic maps from Riemann surfaces
**Tuesday, 10:00**

In my talk (based on joint work with Sebastian Heller and Martin Traizet) I want to introduce a new set of ideas to explicitly constructing harmonic maps from compact Riemann surfaces into 3-space using an implicit function theorem type argument.
When the target space is the round 3-sphere we obtain complete families of high genus embedded constant mean curvature surfaces deforming Lawson's minimal surfaces. Moreover, we compute the Taylor expansion of their area at $g = \infty$ which remarkably relates to values of the Riemann zeta function and multiple-Polylogarithms. Similar ideas for harmonic maps into hyperbolic 3-space leads to more explicitness of the non-abelian Hodge correspondence for the 4-punctured sphere. This correspondence is a real analytic diffeomorphism between the moduli space of Higgs bundles and the moduli space of flat connections. Each space is hereby equipped with a complex structure that is Kähler with respect to the same Riemannian metric. We identify the rescaled limit moduli space of harmonic maps to be the Eguchi-Hanson space of (complex) dimension two and we explicitly compute the limit non-abelian Hodge correspondence as well as its first order derivatives. Also in this case multiple-Polylogarithms appear in the Taylor expansions and the geometry of the problem leads to identities of certain values of multiple-Polylogarithms.

## Tobias Lamm (Karlsruhe)

**Ricci flow of $W^{2,2}$-metrics in four dimensions**
**Tuesday, 15:00**

In this talk we construct solutions to the Ricci-DeTurck flow in four dimensions for $W^{2,2}$-initial metrics. A Ricci-flow related solution is also constructed whose initial value is isometric in a weak sense to the initial value of the Ricci-DeTurck flow. This is a joint work with Miles Simon.

## Christian Lange (Munich)

**From billiards to Alexandrov spaces and Riemannian orbifolds**
**Friday, 10:00**

Billiard flows on convex billiard tables are often only considered away from the corners of the table. In the talk I will discuss and generalize the question when such a billiard flow admits a continuous completion in the context of Alexandrov geometry. In particular, we will see a billiard resp. geodesic flow characterization of alcoves and isosceles tetrahedra, and relate the billiard flow on a sufficiently smooth convex body to the geodesic flow on its boundary. On the way, Riemannian orbifolds will come up naturally. In this context I will also mention some other related but independent results concerning developability conditions and flat metrics.

## Thomas Richard (Paris)

**Small $2$-spheres in manifolds with positive scalar curvature**
**Wednesday, 15:00**

The first results on manifolds with positive scalar curvature where topological in nature and did not yield any control on metric invariants. It was discovered more recently (in dimension 3 by Bray Brendle Neves, in higher dimension by Zhu) that under strong topological assumptions, positive scalar curvature implies the existence of small non contractible 2-spheres. However the simple case of $\mathbb{S}^2 \times \mathbb{S}^2$ remains open. We will show some recent progress in that case.

## Stephan Stadler (MPI Bonn)

**$\mathrm{CAT}(0)$ spaces of higher rank**
**Thursday, 11:20**

A Hadamard manifold – or more generally a CAT(0) space – is said to have higher rank if every geodesic line lies in a flat plane. If a higher rank Hadamard manifold admits finite volume quotients, then it has to be a symmetric space or split as a direct product. This is the content of Ballmann's celebrated Rank Rigidity Theorem, proved in the 80s. It has been conjectured by Ballmann that his theorem generalizes to the synthetic setting of CAT(0) spaces. In the talk I will discuss Ballmann's conjecture and report on recent progress.

## Burkhard Wilking (Münster)

### Positive sectional curvature, torus actions and matroids
### Tuesday, 16:20

This is a report on joint work with Lee Kennard and Michael Wiemeler. Among other things we study positively curved manifold $M$ with an effective isometric action of $T^9$ without finite isotropy groups of even order. We show that these manifolds are rationally equivalent to a rank one symmetric space. I will also explain how a seemingly unrelated new result enters the proof: A finite graph $G$ with an inner metric and first Betti number $b_1(G) = 9$ satisfies $\mathrm{sys}(G) \leq \mathrm{vol}(G)/4$, where $\mathrm{sys}(G)$ is the systole of $G$ and $\mathrm{vol}(G)$ its one dimensional volume.

## Rudolf Zeidler (Münster)

### Distance estimates and rigidity under lower scalar curvature bounds
### Wednesday, 16:20

Abstract Sharp inequalities on Riemannian manifolds subject to lower scalar curvature bounds have become an important aspect in the study of scalar curvature in the recent years, in particular due to a number of new conjectures raised by Gromov. This includes distance estimates on certain cylinders $M \times [-1, 1]$ under a uniform positive lower scalar curvature bound as well as rigidity properties of various model spaces (e.g. Llarull's classical theorem for the sphere: It is impossible to simultaneously increase the area and the scalar curvature on the sphere compared to the standard round metric). In this talk, we will give an introduction to this type of questions and present some recent results, with a slight bias towards methods based on the Dirac operator.