Karlsruhe Institute of Technology

Department of Mathematics
Institute for Algebra and Geometry

# Geometry, Groups and Topology

**September 12th – 15th, 2011**

## Organizers

Anand Dessai (Fribourg)

Enrico Leuzinger (Karlsruhe)

Wilderich Tuschmann (Karlsruhe)

# Program

All talks take place in the ‘Redtenbacher-Hörsaal’, Room 050, Building 10.91, Engelbert-Arnold-Str. 4.

## Monday

**$10^{00}$ – $13^{00}$** Registration in Room 4A-09, ‘Allianzgebäude’ (Building 05.20), Kaiserstr. 89-93

**$13^{15}$ – $14^{00}$** Registration at the entrance of the ‘Redtenbacher-Hörsaal’

**$14^{00}$ – $15^{00}$** **Matthias Kreck** (Bonn)
*Equivariant Poincaré Duality*

**$15^{00}$ – $16^{00}$** Coffee break at the ‘Gastdozentenhaus’

**$16^{00}$ – $17^{00}$** **Bernhard Hanke** (Augsburg)
*Large and small group homology*

## Tuesday

**$9^{30}$ – $10^{30}$** **Luis Guijarro** (Madrid)
*The cut locus and balanced split loci in Riemannian manifolds*

**$10^{30}$ – $11^{00}$** Coffee break

**$11^{00}$ – $12^{00}$** **Uwe Semmelmann** (Stuttgart)
*Almost complex structures on quaternion Kähler manifolds and inner symmetric spaces*

**$12^{00}$ – $14^{00}$** Lunch break

**$14^{00}$ – $15^{00}$** **Lizhen Ji** (Ann Arbor)
*Geometry and analysis of moduli spaces of Riemann surfaces*

**$15^{00}$ – $16^{00}$** Coffee break at the ‘Gastdozentenhaus’

**$16^{00}$ – $17^{00}$** **Hugo Parlier** (Fribourg)
*The geometry of surfaces of large genus*

## Wednesday

| | |
|---|---|
| **$9^{30}$ – $10^{30}$** | **Oliver Baues** (Karlsruhe)<br>*Fibrations of aspherical Riemannian manifolds* |
| **$10^{30}$ – $11^{00}$** | Coffee break |
| **$11^{00}$ – $12^{00}$** | **Richard Weidmann** (Kiel)<br>*Makanin-Razborov diagrams for hyperbolic groups* |
| **$12^{00}$ – $14^{00}$** | Lunch break |
| **$14^{00}$ – $15^{00}$** | **Michelle Bucher-Karlsson** (Genf)<br>*Euler Characteristic, Simplicial volume and the Schläfli volume formula* |
| **$15^{00}$ – $16^{00}$** | Coffee break at the ‘Gastdozentenhaus’ |
| **$16^{00}$ – $17^{00}$** | **Gabriela Weitze-Schmithüsen** (Karlsruhe)<br>*Origamis in Teichmüller space* |
| **$19^{00}$** | Conference Dinner at the ‘Gastdozentenhaus’ |

## Thursday

| | |
|---|---|
| **$9^{30}$ – $10^{30}$** | **Alexander Lytchak** (Münster)<br>*Spherical Buildings and Polar Foliations* |
| **$10^{45}$ – $11^{45}$** | **Bernd Ammann** (Regensburg)<br>*Lower bounds for the Yamabe invariant* |
| **$11^{45}$ – $12^{15}$** | Coffee break |
| **$12^{15}$ – $13^{15}$** | **Burkhard Wilking** (Münster)<br>*Applications of the Margulis Lemma* |
| **$15^{00}$** | Guided tour at Karlsruhe’s Center for Art and Media ‘ZKM’ |

# Abstracts

**Bernd Ammann** (Regensburg)

*Lower bounds for the Yamabe invariant*

In this talk I want to give an overview over some old and new results about the (smooth) Yamabe invariant in joint work with M. Dahl and E. Humbert. We will see for example that the Yamabe invariant of a simply connected compact spin manifold of dimension $5$ is between $45$ and $79$. Similar estimates hold for $2$-connected compact spin manifolds with vanishing index.

Let us give some more details. The conformal Yamabe constant of a compact riemannian manifold $(M, g_0)$ is defined as

$$Y(M, [g_0]) := \inf \int_M \mathrm{scal}^g \, dv^g$$

where the infimum runs over all metrics $g$ of volume $1$ in $[g_0]$. The smooth Yamabe invariant of $M$ is then defined as

$$\sigma(M) := \sup Y(M, [g_0])$$

where the supremum runs over all conformal classes $[g_0]$ on $M$.

These invariants are tightly related to the existence of metrics of constant scalar curvature in a given conformal class. The invariant $\sigma(M)$ is positive iff $M$ carries a metric of positive scalar curvature.

We have proven a formula that estimates the behaviour of $\sigma(M)$ under performing surgery at $M$, namely if $N$ is obtained by surgery of codimension $k \geq 3$ from $M$, then

$$\sigma(N) \geq \min\{\sigma(M), \Lambda_{n,k}\},$$

where $\Lambda_{n,k} > 0$ only depends on $n = \dim M$ and $k$.

The constants $\Lambda_{n,k} > 0$ arise as conformal Yamabe constants of certain limit spaces which are products of rescaled spheres with the standard hyperbolic spaces.

In recent work we found an efficient method to control the Yamabe constants of products spaces provided that both factors are of dimension at least $3$. This formula yields positive lower bounds for $\Lambda_{n,k}$ in the case $k \notin \{1, n-3\}$. We also found a method to compare the conformal Yamabe constant of our model spaces with the conformal Yamabe invariant of spaces like $\mathbb{R}^2 \times S^2$, $\mathbb{R}^3 \times S^2$ and $\mathbb{R}^2 \times S^3$. The Yamabe constants of the latter spaces were recently calculated by Petean and Ruiz. This method is e.g. sufficient to control $\Lambda_{5,1}$ and $\Lambda_{5,2}$ and thus yields the explicit bound mentioned above for $5$-manifolds.

**Michelle Bucher-Karlsson** (Genf)

*Euler Characteristic, Simplicial volume and the Schläfli volume formula*

I will consider two topological invariants of manifolds, the classical Euler characteristic and the newer simplicial volume (introduced by Gromov in 1980), and discuss their analogies and differences. In particular, I will discuss a question of Gromov, asking if the vanishing of the simplicial volume of an aspherical manifold implies the vanishing of its Euler characteristic. Based on a generalization of the classical Schläfli volume formula, I will answer this question for a modified version of the simplicial volume, defined for immersed simplices.

**Oliver Baues** (Karlsruhe)

*Fibrations of aspherical Riemannian manifolds*

Every compact aspherical Riemannian manifold admits a canonical series of fibrations which are Riemannian orbibundles with infrasolv-fibers. Each step of the resulting tower of maps arises from the action of the continuous part of the isometry group on the universal cover. The length of the tower and the geometry of its base measure the degree of continuous symmetry of an aspherical Riemannian manifold. The symmetry is called large if the base is locally symmetric. We show that closed aspherical manifolds with exotic smooth structure do not support a metric with large symmetry.

**Luis Guijarro** (Madrid)

*The cut locus and balanced split loci in Riemannian manifolds*

We will review recent results on the cut loci of points or hypersurfaces in a Riemannian manifold. More precisely, we characterize the cut locus as a *balanced split locus*; conversely, we examine what other possible balanced split loci exist for a given metric. This is joint work with Pablo Angulo-Ardoy (UAM).

**Bernhard Hanke** (Augsburg)

*Large and small group homology*

For several instances of metric largeness like enlargeability or having hyperspherical universal covers, we construct non-large vector subspaces in the rational homology of finitely generated groups. The functorial properties of this construction imply that the corresponding largeness properties of closed manifolds depend only on the image of their fundamental classes under the classifying map. This is applied to construct examples of essential manifolds whose universal covers are not hyperspherical, thus answering a question of Gromov (1986), and, more generally, essential manifolds which are not enlargeable.

**Lizhen Ji** (Ann Arbor)

*Geometry and analysis of moduli spaces of Riemann surfaces*

In this talk, I will describe several results on the Riemannian geometry and spectral theory of moduli spaces of Riemann surfaces.

**Matthias Kreck** (Bonn)

*Equivariant Poincaré Duality*

Ordinary equivariant (co)homology is usually defined via the Borel construction. Poincaré duality fails as much as it can. Using stratifolds one can define a new cohomology theory which is Poincaré dual to ordinary equivariant homology and similarly a new homology theory which is Poincaré dual to ordinary cohomology. In a similar spirit one can define Bredon-type (co)homology theories which might be useful for equivariant characteristic classes. I hope to demonstrate the use of these theories in some examples.

**Alexander Lytchak** (Münster)

*Spherical Buildings and Polar Foliations*

Spherical buildings are rigid combinatorial objects introduced by Tits as an important tool in the study of algebraic groups. Due to their rigid nature, it does not happen often that a building appears in an a priori non-algebraic situation, and every time it happens, the consequences are very strong. In my talk, I will discuss a few applications of spherical buildings in Riemannian geometry.

**Hugo Parlier** (Fribourg)

*The geometry of surfaces of large genus*

What can a surface of large genus look like? What does a typical or random surface look like? In certain contexts, the talk will be about studying the behavior of curves on surfaces to understand the geometry of surfaces and their related moduli spaces. In the opposite direction, I'll discuss some work with Guth and Young about what we can learn about random surfaces by studying their moduli spaces. Various parts of the talk are joint with W. Cavendish, with F. Balacheff and S. Sabourau, and with L. Guth and R. Young.

**Uwe Semmelmann** (Stuttgart)

*Almost complex structures on quaternion Kähler manifolds and inner symmetric spaces*

It is a classical result of Hirzebruch that the quaternion projective spaces do not admit any almost complex structure. In my talk I will show how the Atiyah-Singer index theorem can be used to give a short alternative proof, which applies to a much larger class of manifolds. With a similar idea it is possible to decide which inner symmetric spaces admit almost complex structures.

**Richard Weidmann** (Kiel)

*Makanin-Razborov diagrams for hyperbolic groups*

For an arbitrary finitely presented group $G$ and a free group $F$, the set $\mathrm{Hom}(G,F)$ was described by Razborov relying on earlier work of Makanin. This description was refined by Kharlampovich and Myasnikov and independently by Sela. Sela further generalized his approach to describe $\mathrm{Hom}(G,\Gamma)$ for an arbitrary word-hyperbolic group $\Gamma$. A full understanding of $\mathrm{Hom}(G,\Gamma)$ is one of the basic building blocks for understanding the elementary theory of $\Gamma$. It is the aim of this talk to outline Sela's approach to Makanin-Razborov diagrams; incremental contributions by the speaker and Reinfeldt are discussed if time permits.

**Gabriela Weitze-Schmithüsen** (Karlsruhe)

*Origamis in Teichmüller space*

Teichmüller disks are isometric and holomorphic images of the Poincaré upper half-plane in Teichmüller space (endowed with the Teichmüller metric). There is a special interest on those, whose image in moduli space is an algebraic complex curve, in this case called Teichmüller curve. Teichmüller disks are obtained by a handy construction using translation surfaces. We study examples called square-tiled surfaces or origamis that arise from gluing finitely many copies of the Euclidean unit square along their edges. These surfaces can be represented by simple combinatorial data. We explain how the corresponding Teichmüller disks are also naturally embedded into Culler-Vogtmann outer space $CV_n$, the classification space of marked metric graphs of genus $n$, and give applications of this.

**Burkhard Wilking** (Münster)

*Applications of the Margulis Lemma*

We (joint work with V. Kapovitch) establish a Margulis Lemma for manifolds with lower Ricci curvature bound. In this talk I will focus on some of the applications of the Margulis Lemma rather than the proof of it.