On-Line Coloring Between Two Lines

Stefan Felsner
Technische Universität Berlin

Piotr Micek
Jagiellonian University Kraków

Torsten Ueckerdt
Karlsruhe Institute of Technology

Karlsruhe Institute of Technology

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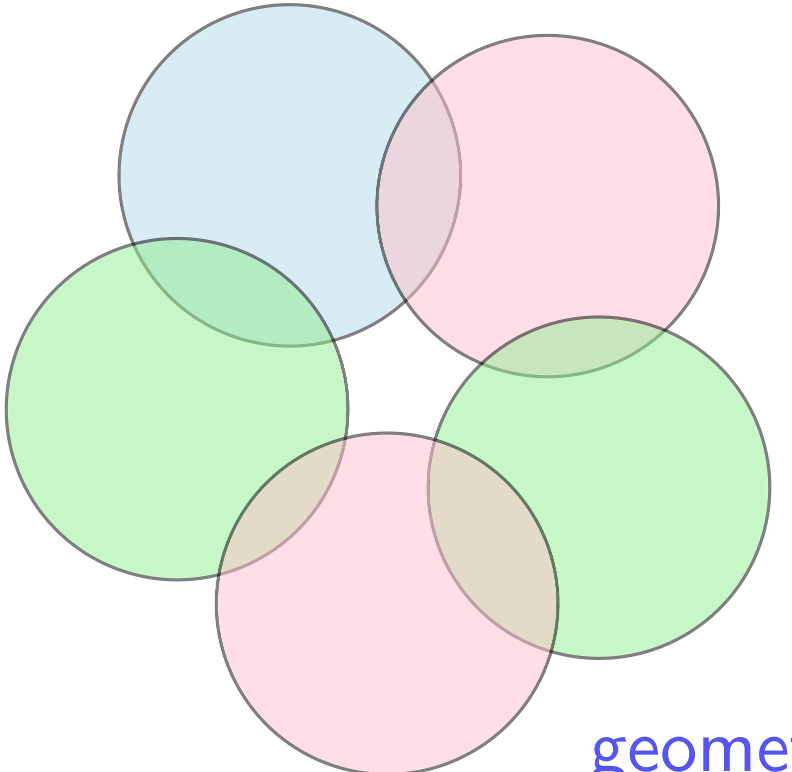
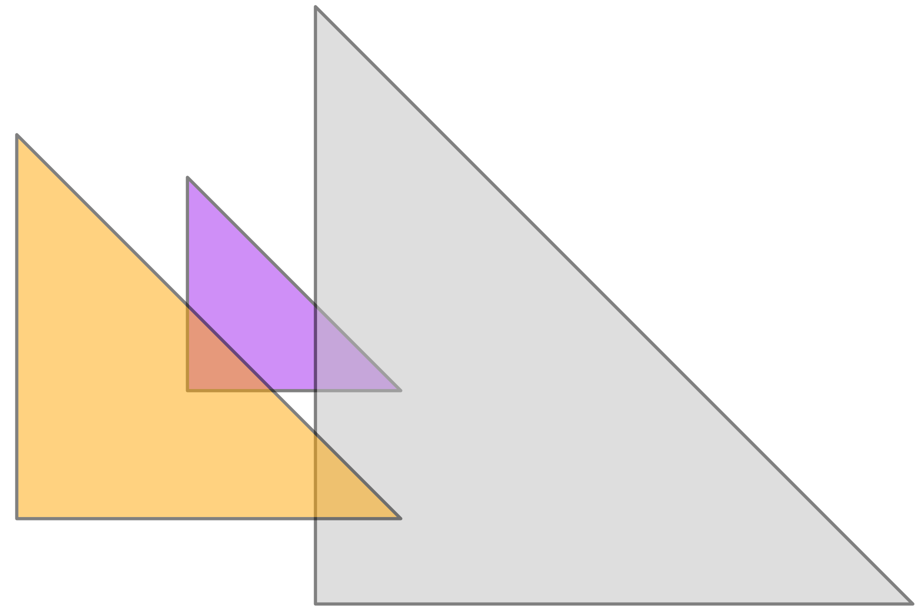
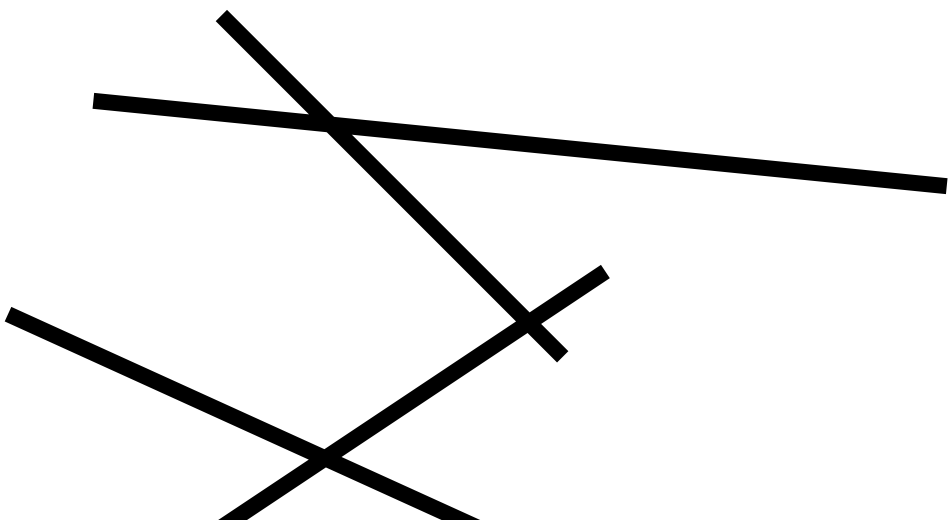
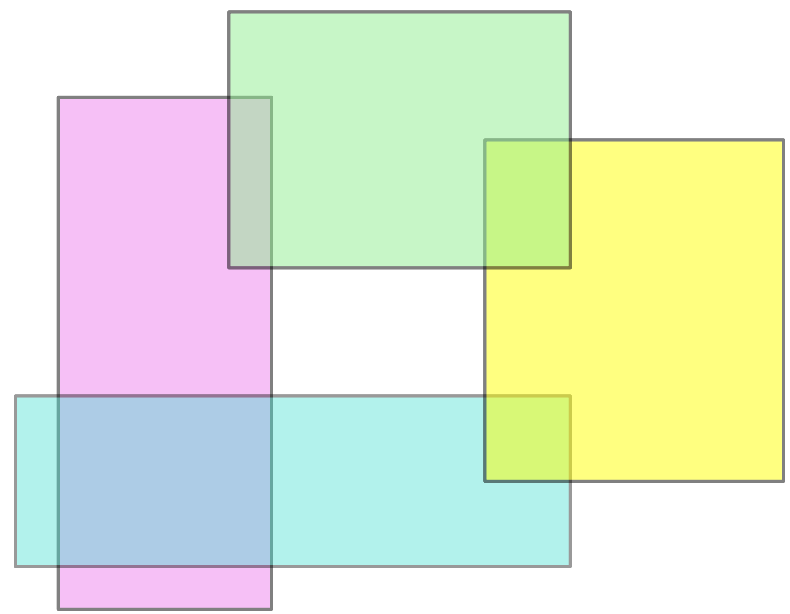
Karlsruhe



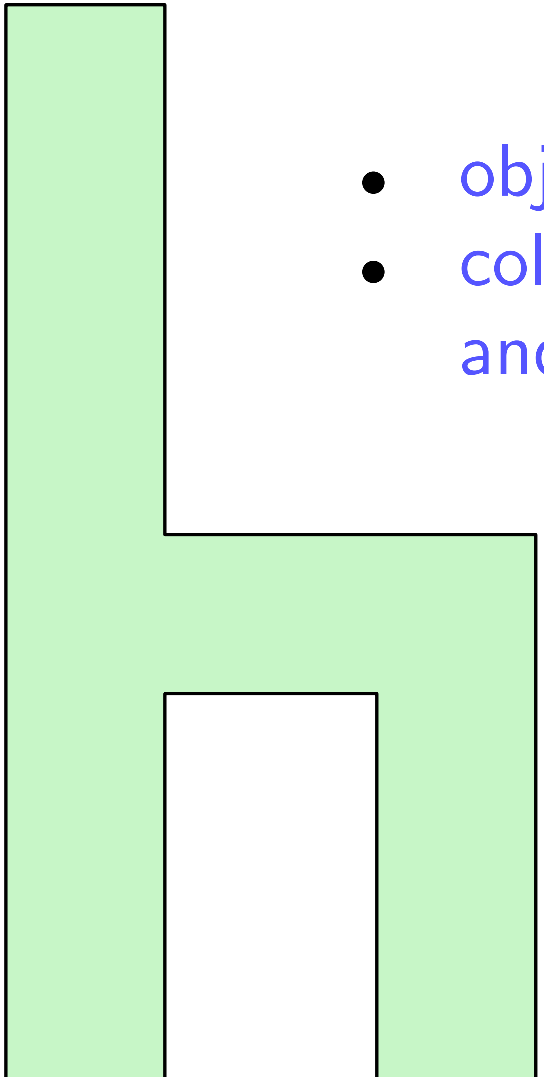


geometric intersection graphs

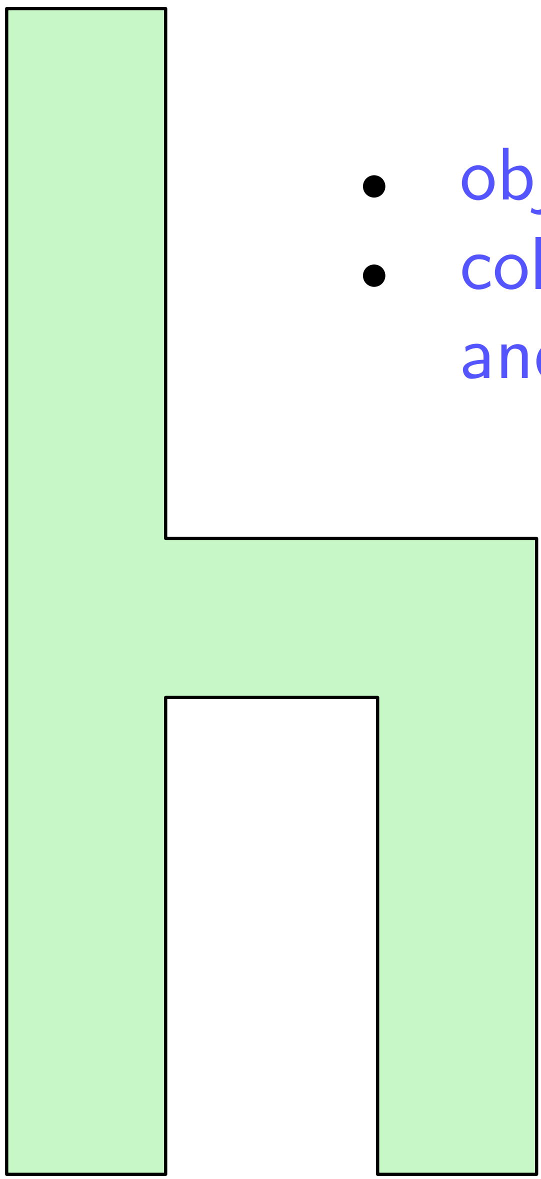




on-line coloring



- objects are created one at a time
- colors are assigned immediately and irrevocably



on-line coloring

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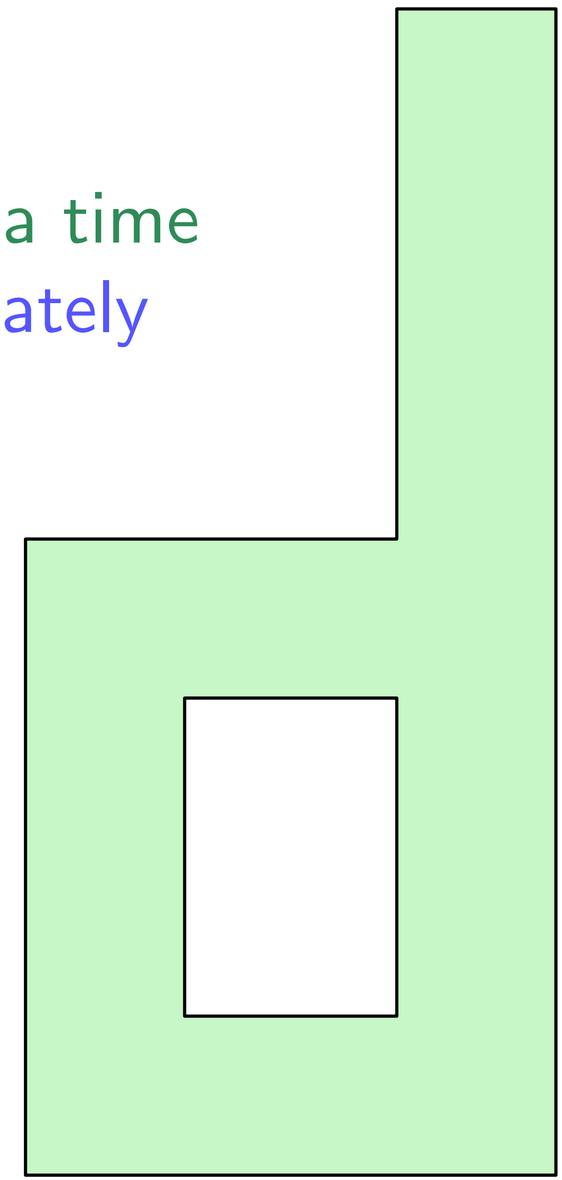

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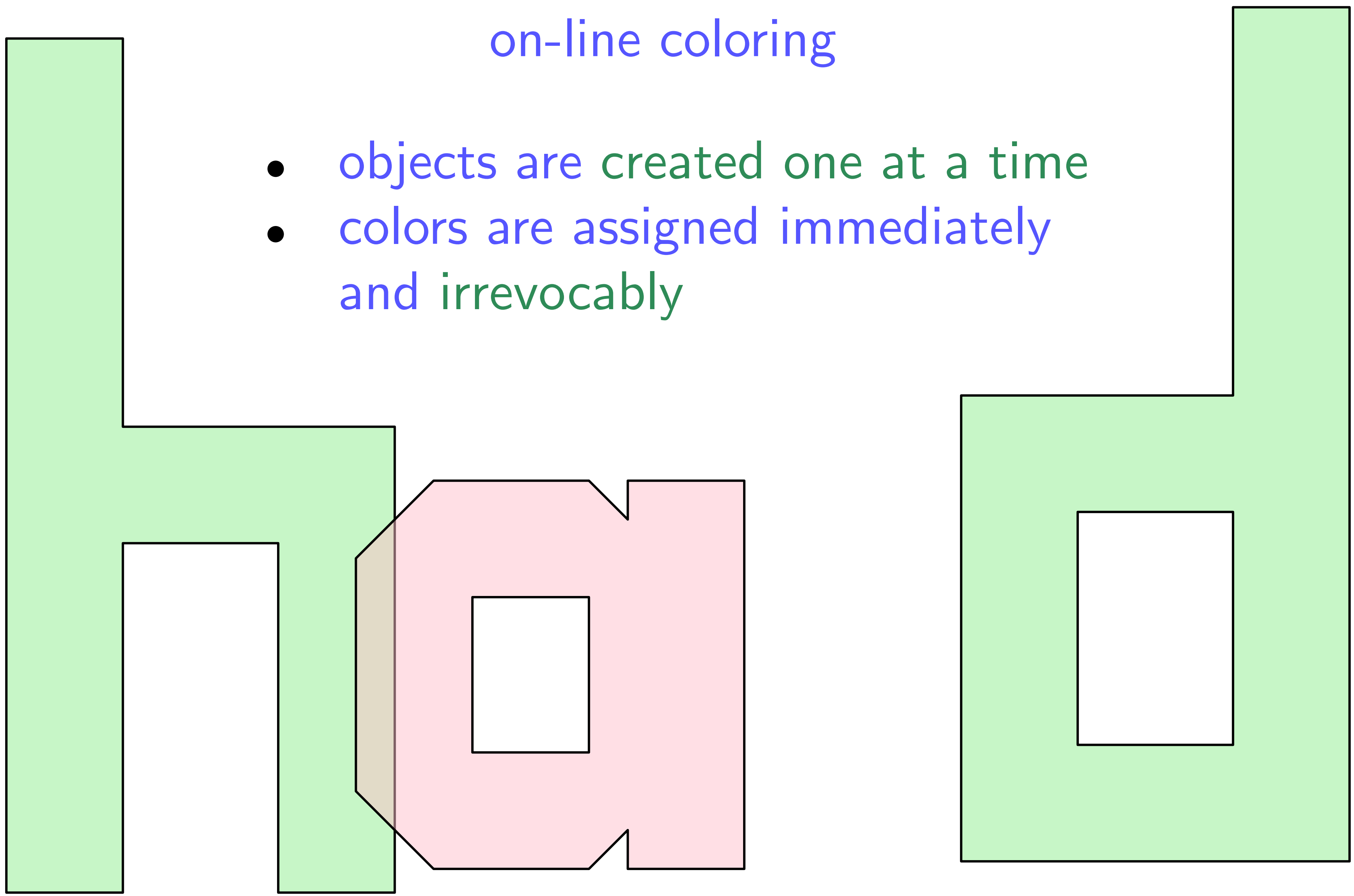

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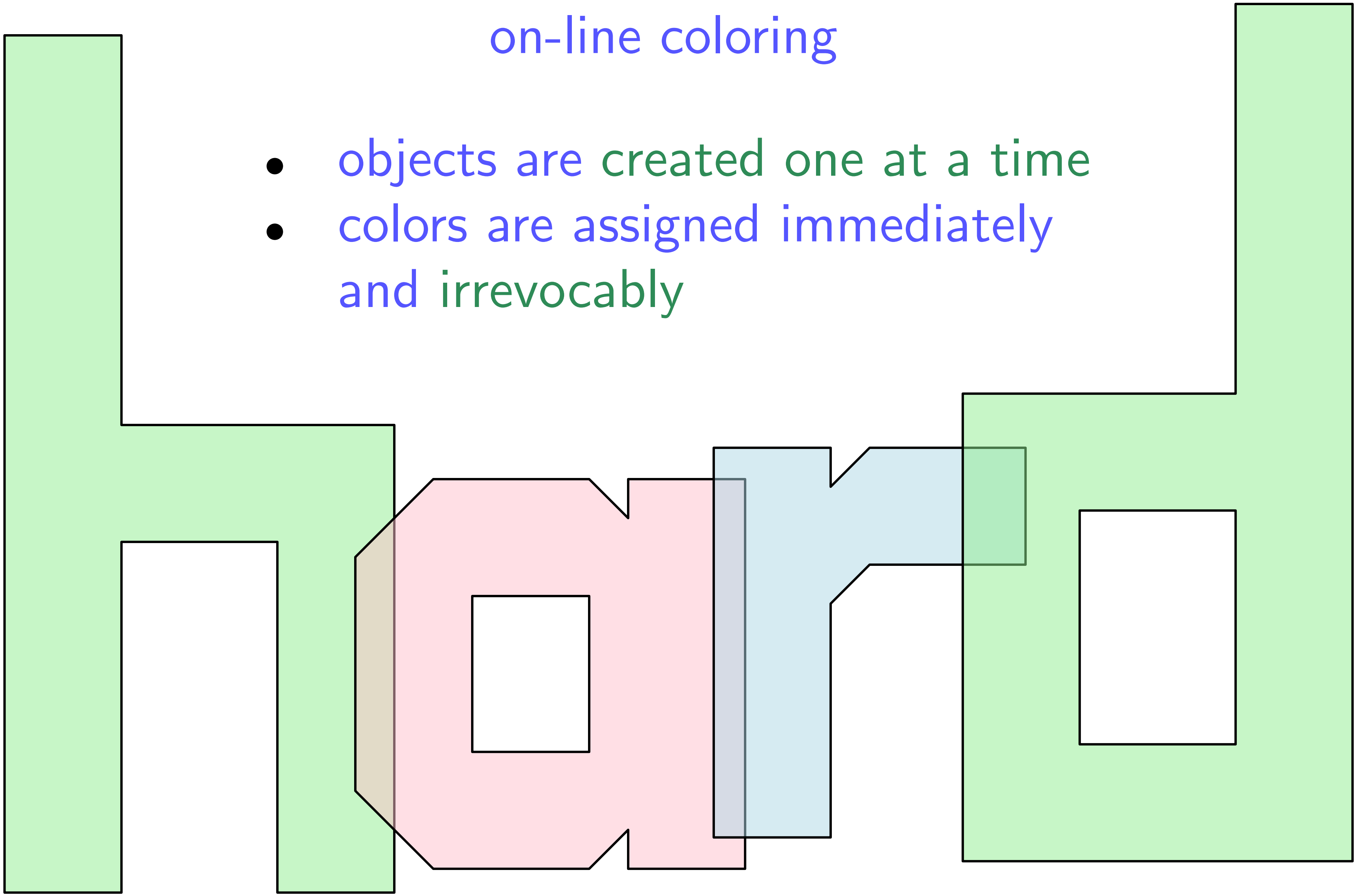


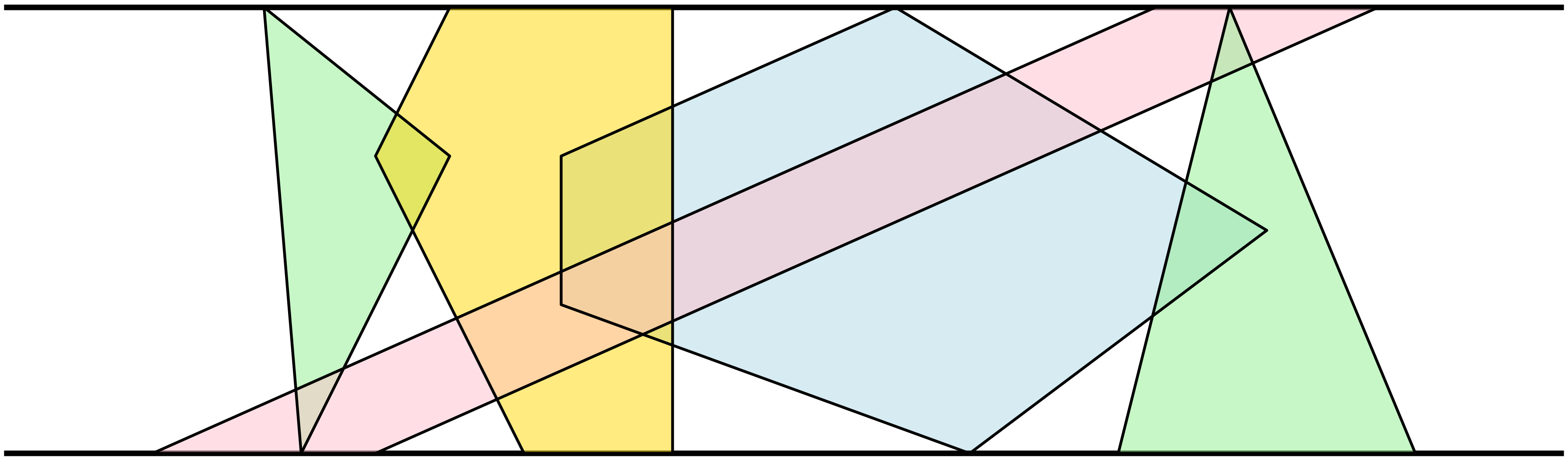
Part 1

convex sets between two lines

Part 2

more new classes

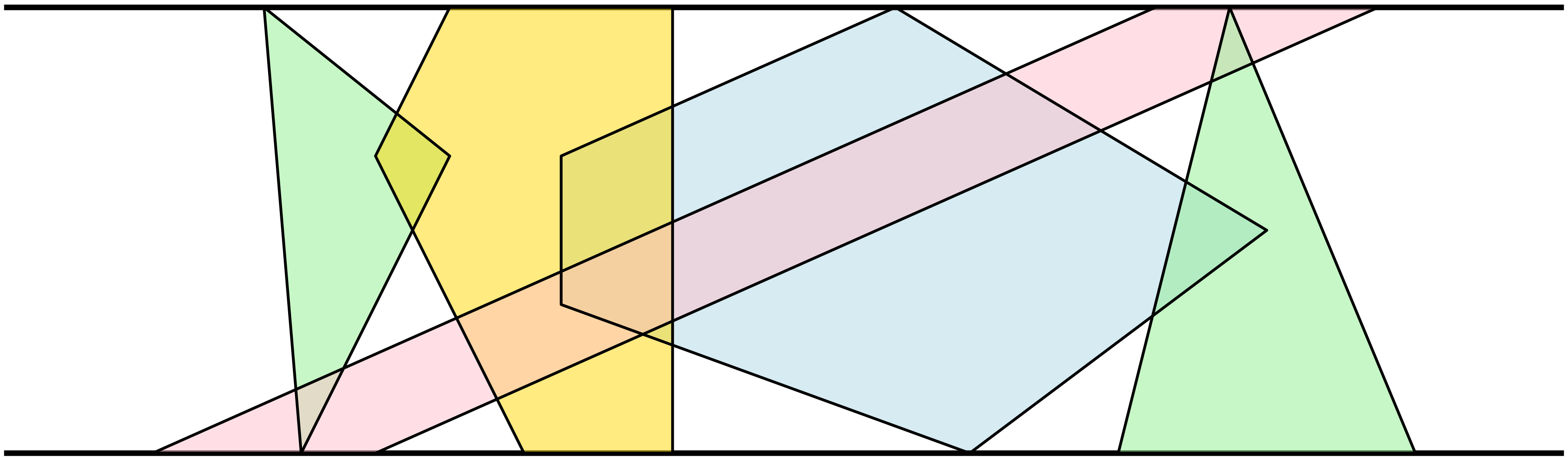
- intersection graphs of convex compact objects between two parallel lines

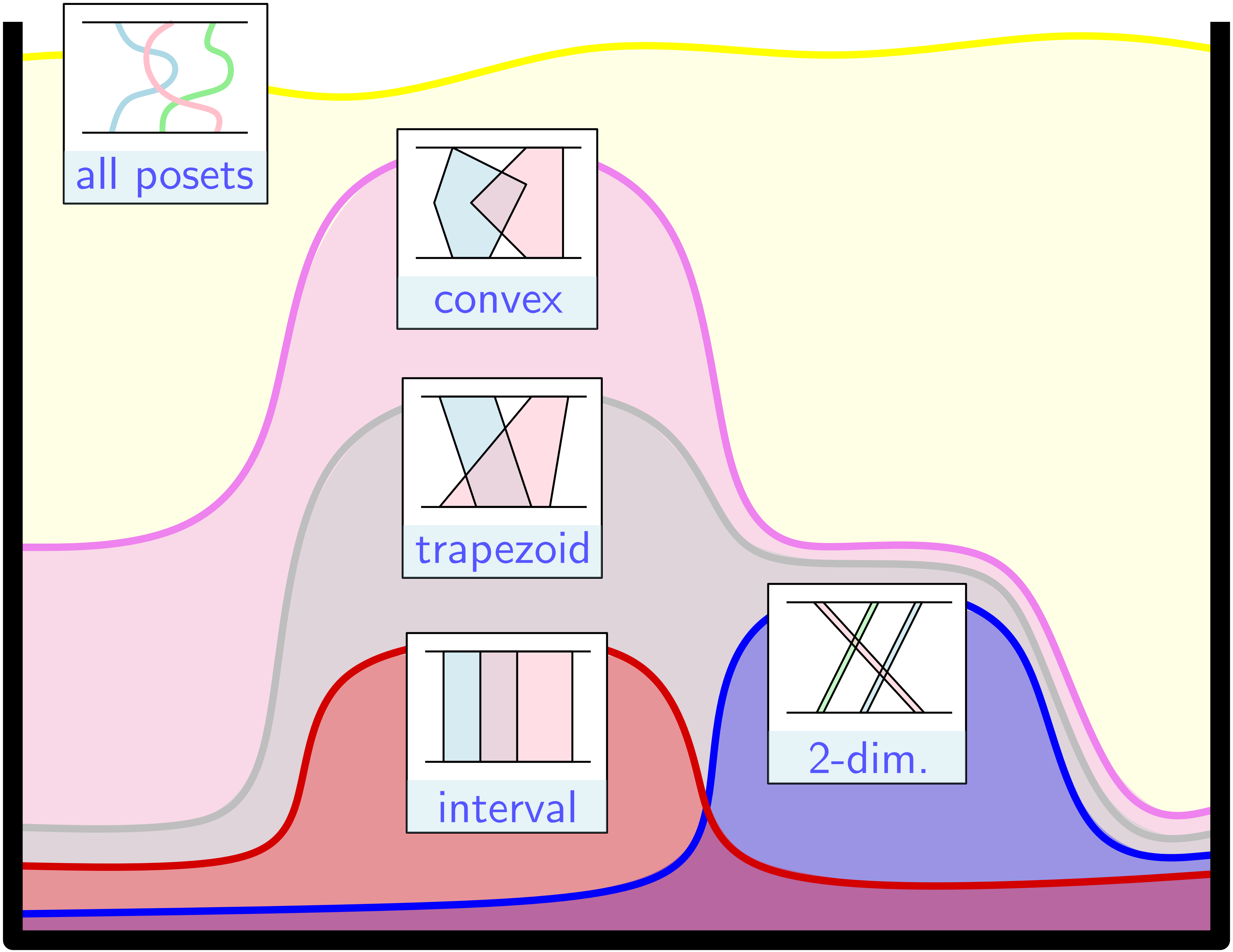


- generalize interval graphs, permutation graphs, bounded tolerance graphs, . . .

- subclass of cocomparability graphs $\longrightarrow \chi = \omega$

  ↳ χ = chromatic number, ω = maximum clique size

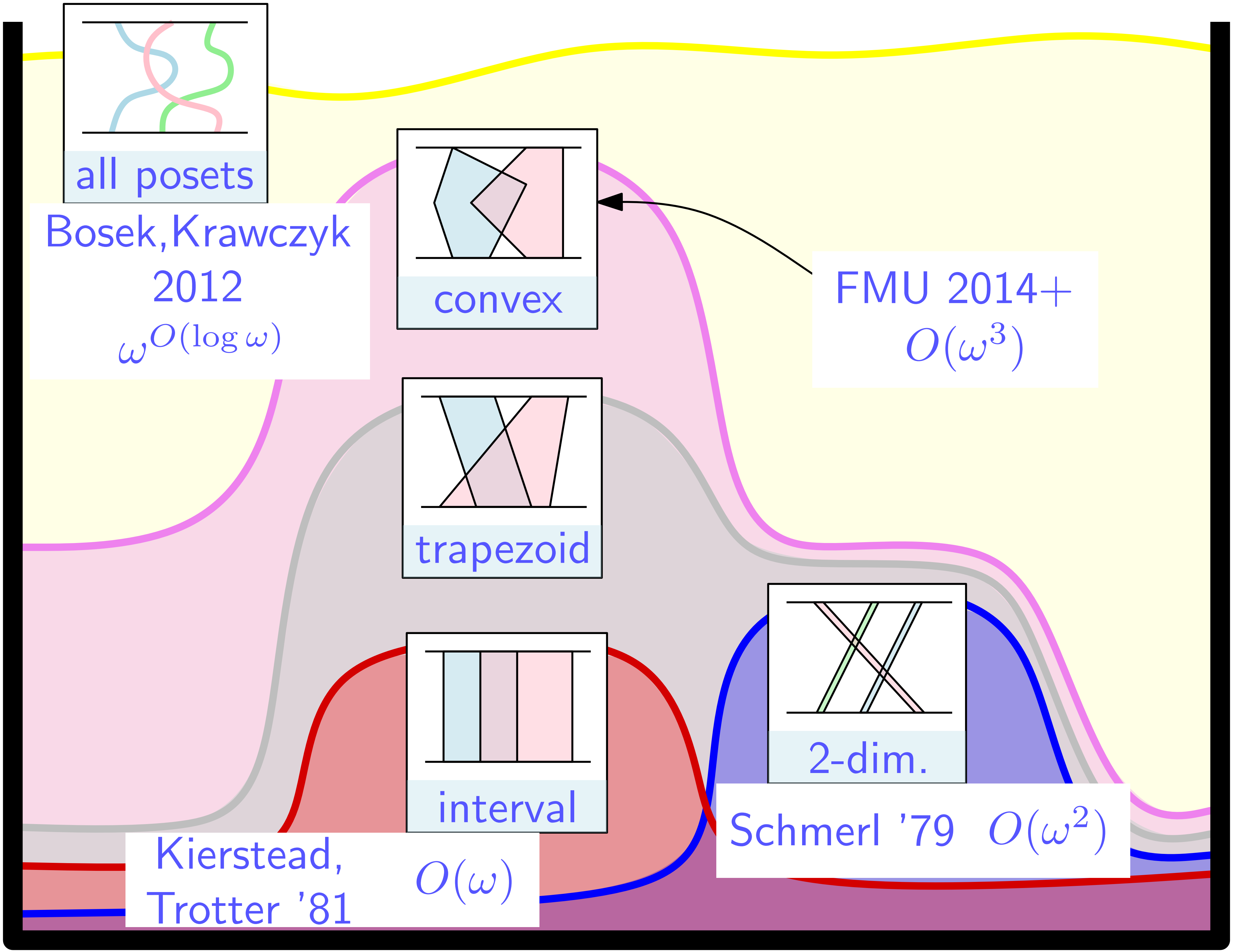



all posets
convex
trapezoid
interval
2-dim.

- subclass of cocomparability graphs $\longrightarrow \chi = \omega$

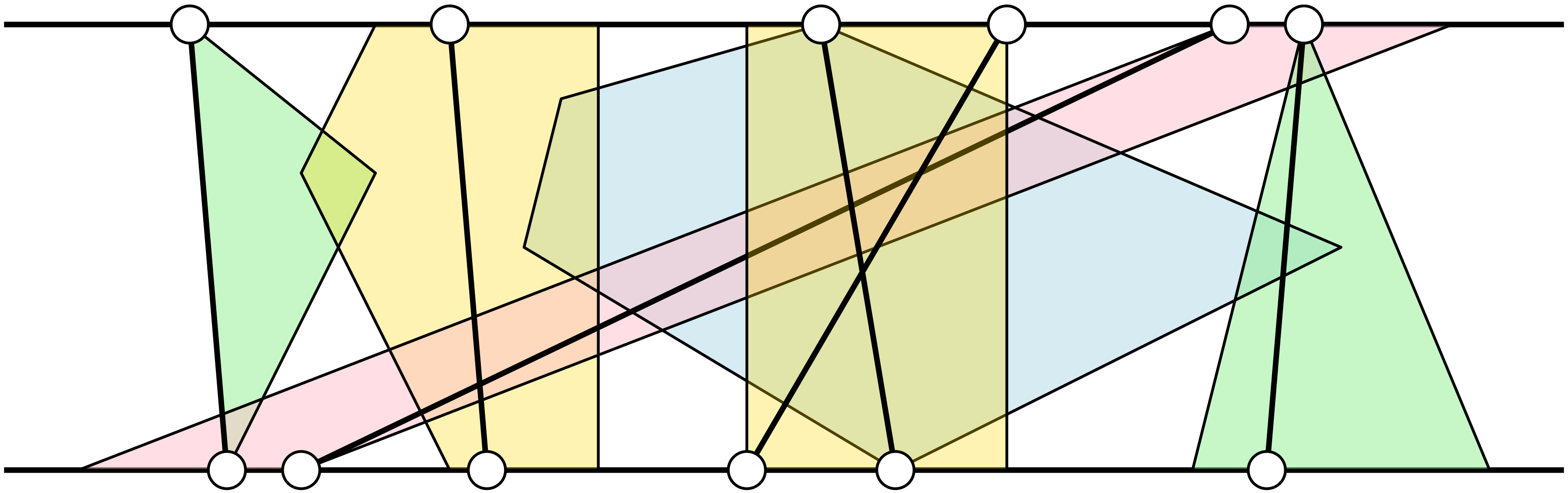
  ↳ χ = chromatic number, ω = maximum clique size

Thm. Intersection graphs of convex objects between two lines can be on-line colored with $O(\omega^3)$ colors.


all posets
Bosek,Krawczyk
2012
$\omega^{O(\log \omega)}$
convex
FMU 2014+
$O(\omega^3)$
trapezoid
2-dim.
interval
Schmerl '79 $O(\omega^2)$
Kierstead,
Trotter '81
$O(\omega)$

Proof idea.

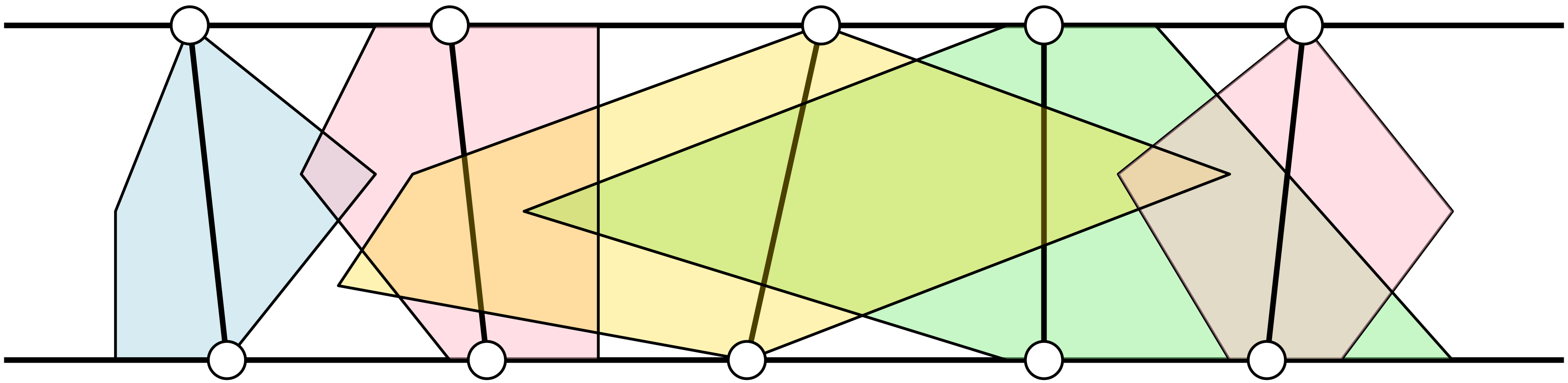
- “base” $=$ segment between attachment points



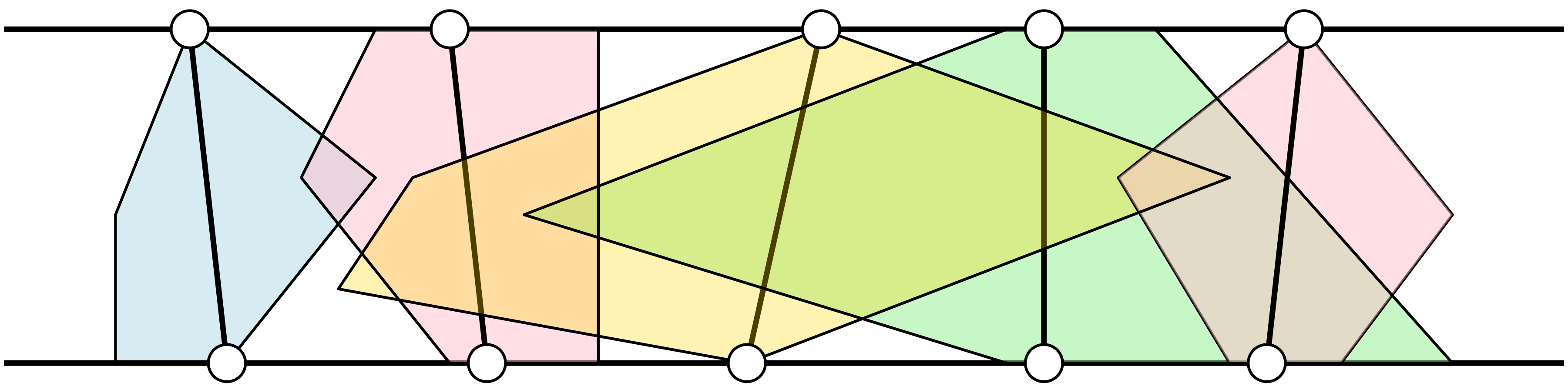
- $\begin{Bmatrix}\alpha\\ \beta\end{Bmatrix}$ = maximum size of $\begin{Bmatrix}\text{clockwise}\\ \text{counterclockwise}\end{Bmatrix}$ clique

- “color” $= (\alpha, \beta, \gamma \to$ to be defined$)$

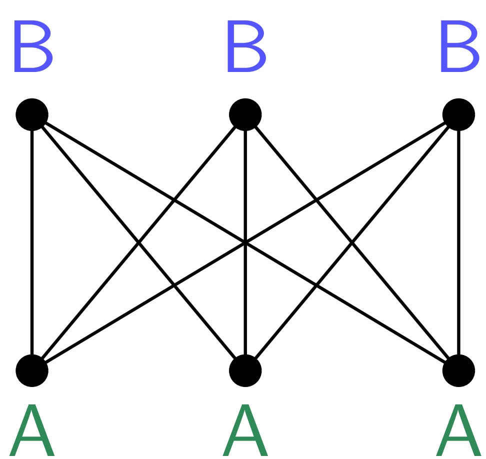
- within an $(\alpha, \beta, \quad)$-class base segments are disjoint

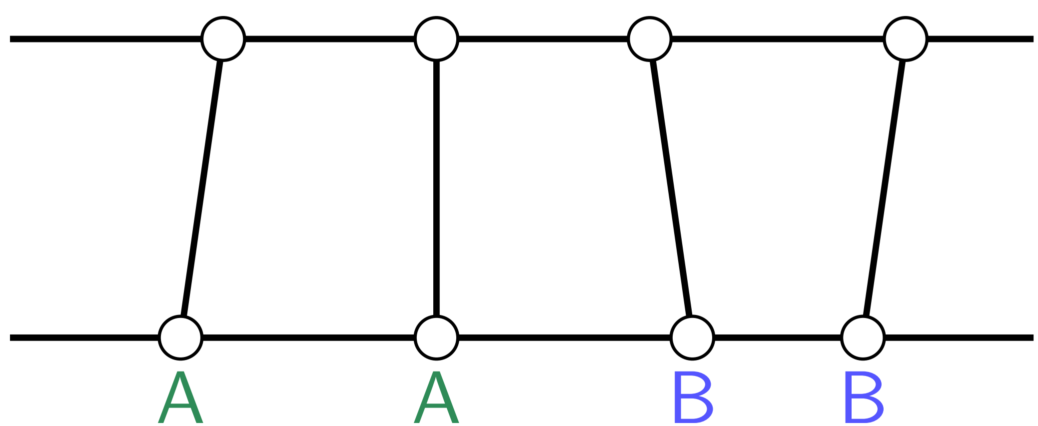


- within an $(\alpha, \beta, \ )$-class base segments are disjoint



Lem. Every $(\alpha, \beta, \ )$-class is $K_{3,3}$-free.





Lem. Every $(\alpha, \beta, \ \ )$-class is $K_{3,3}$-free.

- $\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix}$ = maximum size of $\begin{Bmatrix} \text{clockwise} \\ \text{counterclockwise} \end{Bmatrix}$ clique

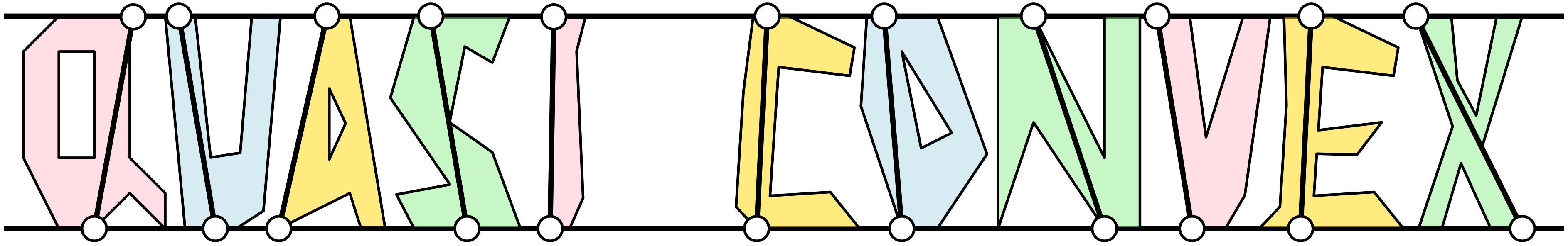
⟶ $\#(\alpha, \beta, \ \ )$-classes $\leq \omega^2$

Thm. (Dujmović, Joret, Wood 2012)
$K_{3,3}$-free cocomparability graphs can be on-line colored with $O(\omega)$ colors.

⟶ $\#$ colors $= O(\omega) \cdot \#(\alpha, \beta, \ \ )$-classes $= O(\omega^3)$

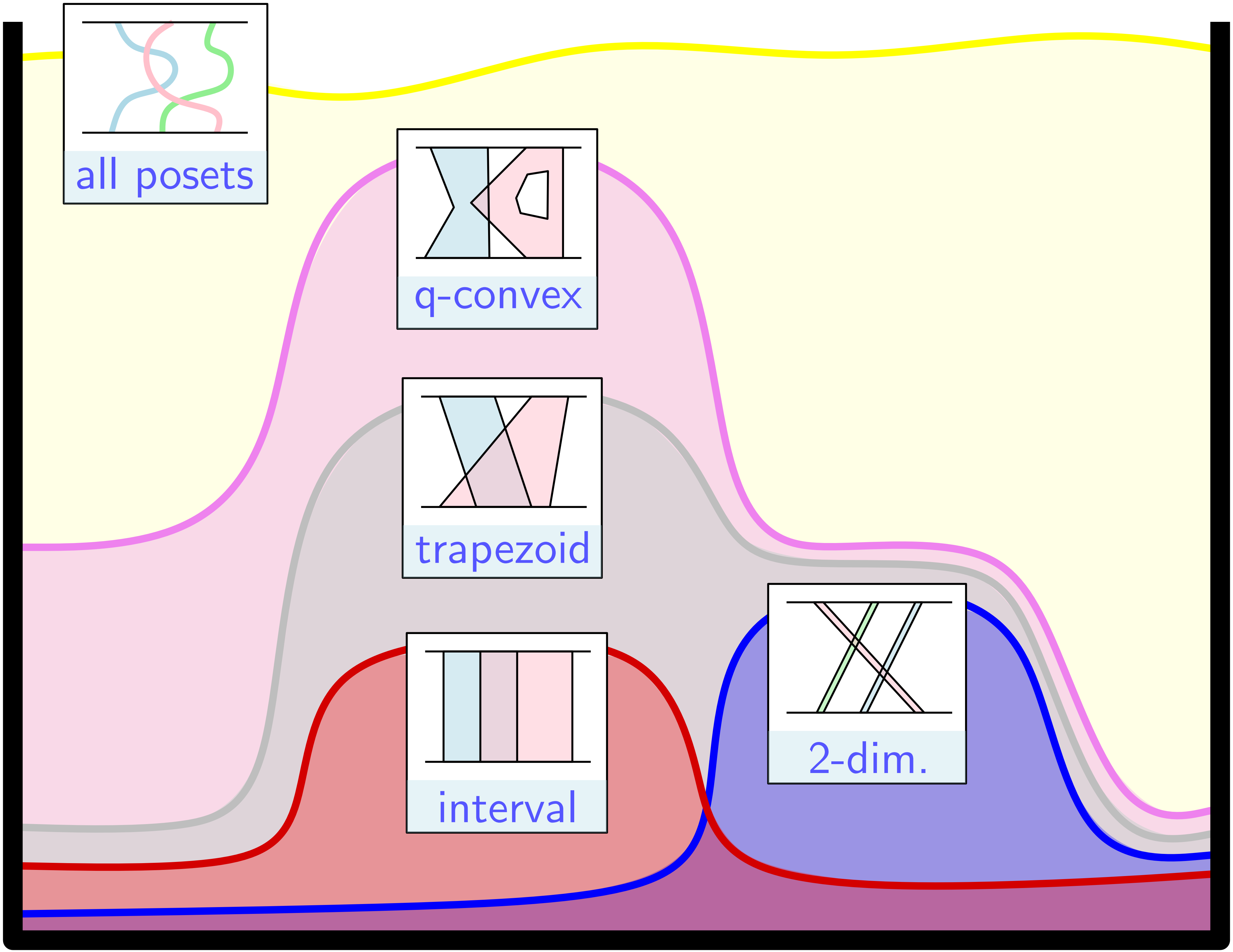
□

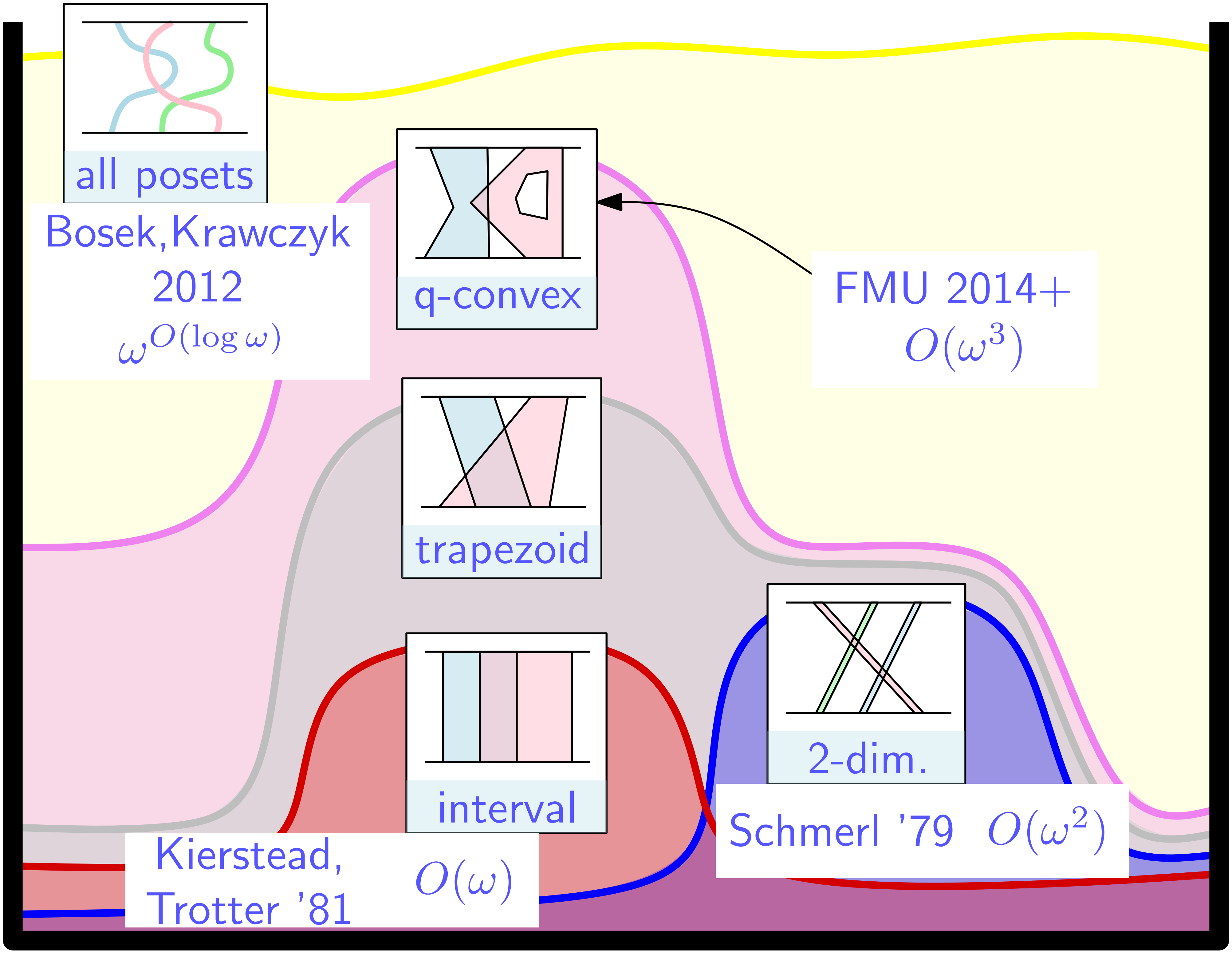
We have proven something stronger . . .



Thm. Intersection graphs of quasi-convex objects between two lines can be on-line colored with $O(\omega^3)$ colors.

Lem. Some “quasi-convex” posets are “not convex”.


all posets
q-convex
trapezoid
interval
2-dim.


all posets
Bosek,Krawczyk
2012
$\omega^{O(\log \omega)}$
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$O(\omega)$

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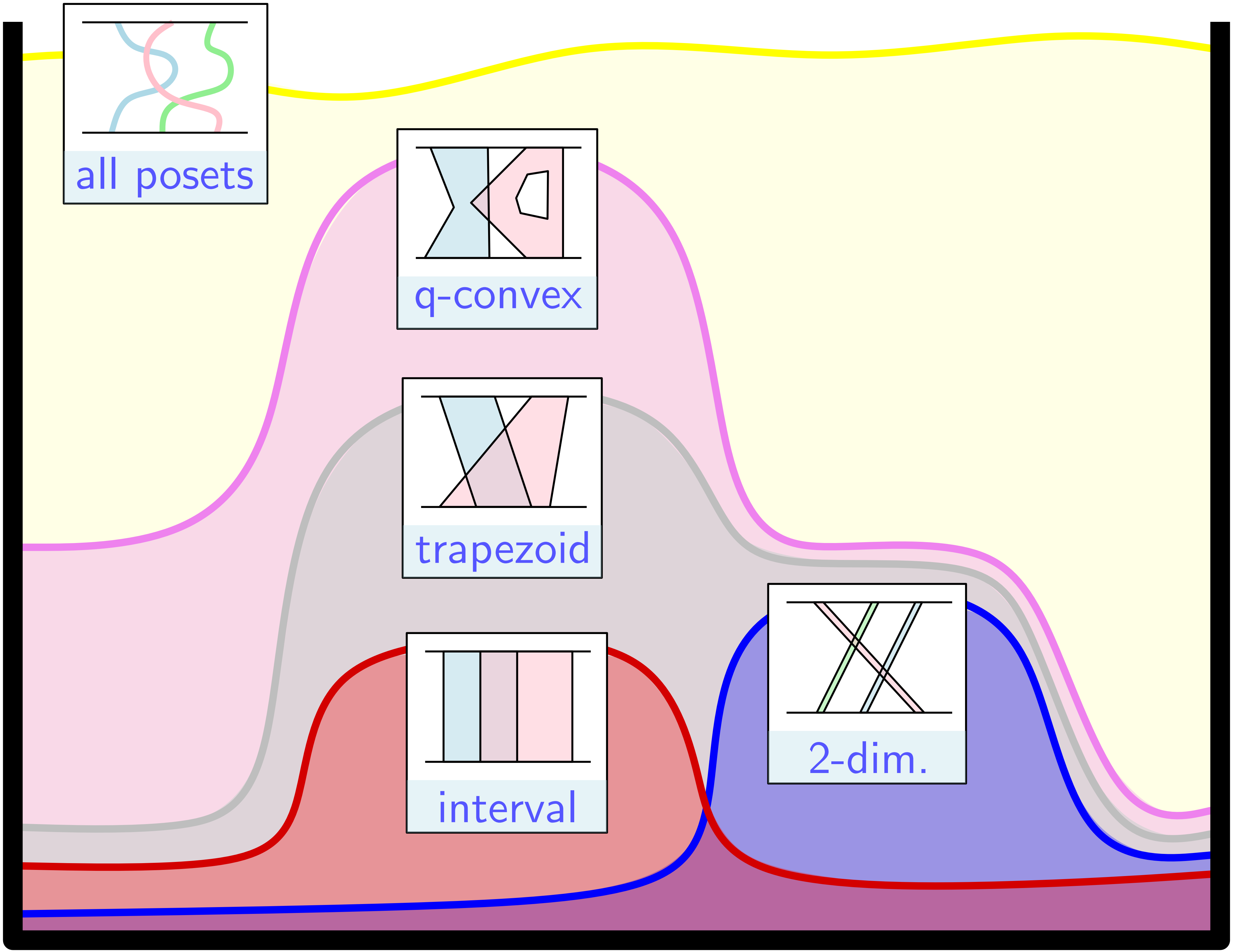
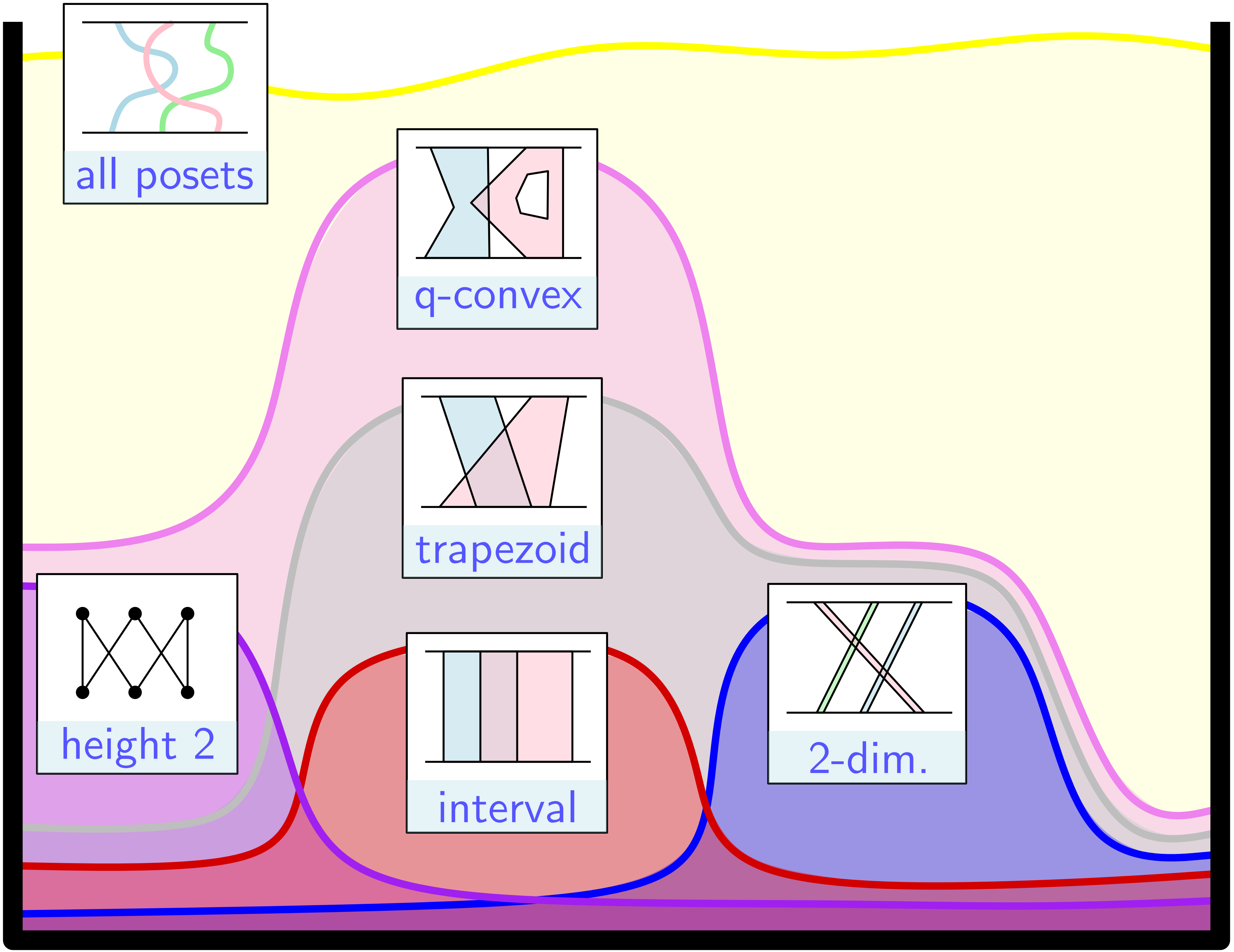
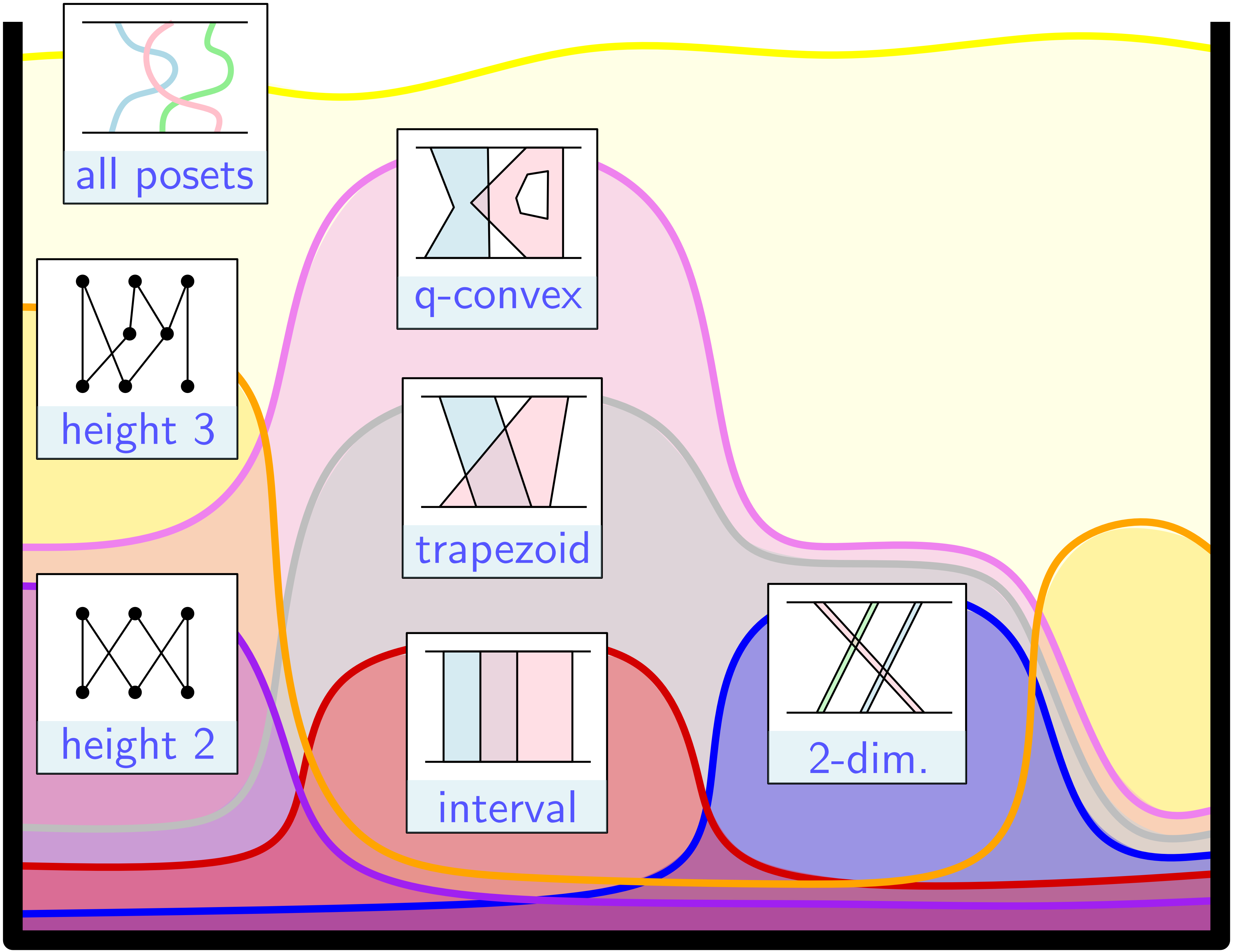
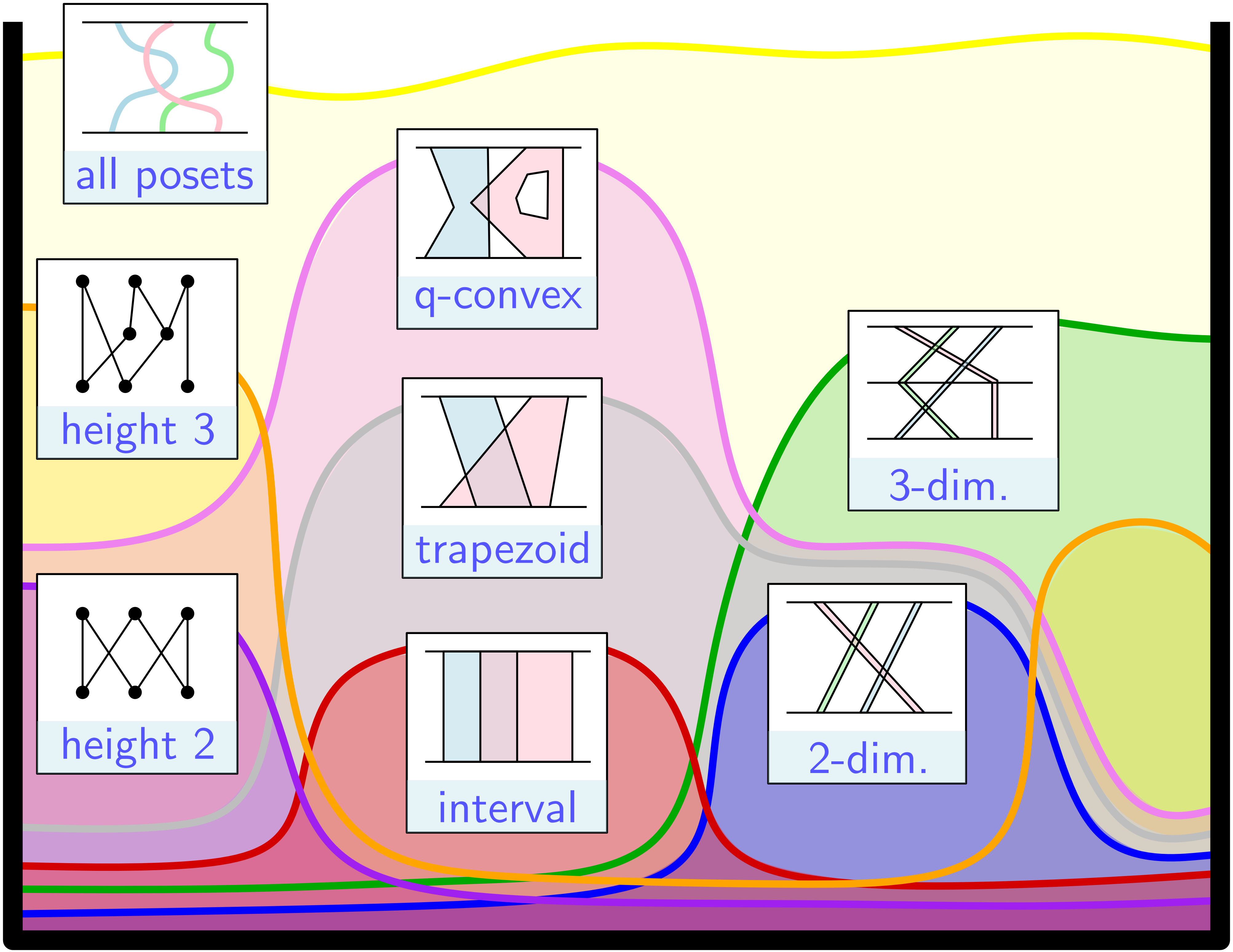


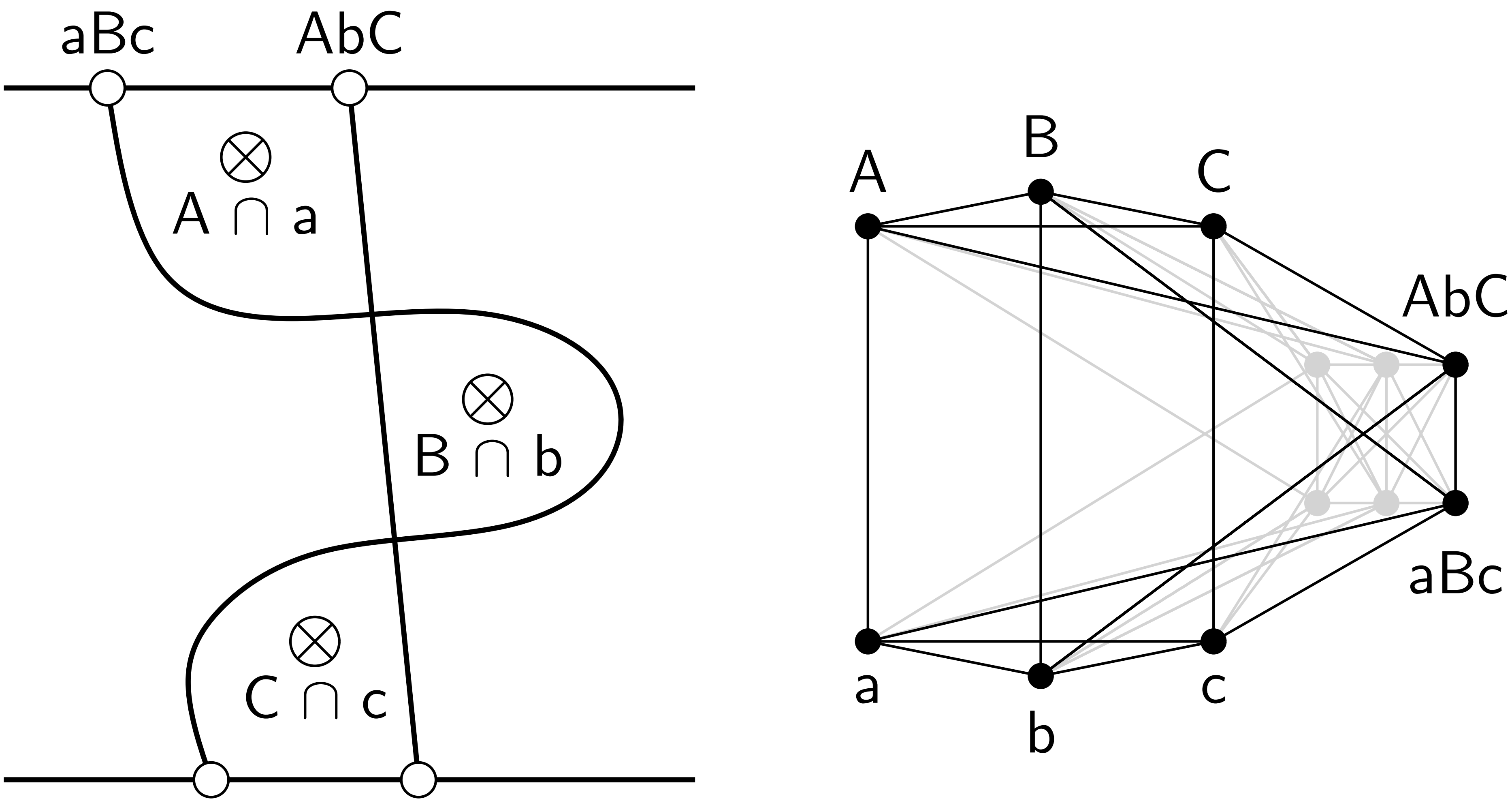
all posets
q-convex
trapezoid
interval
2-dim.

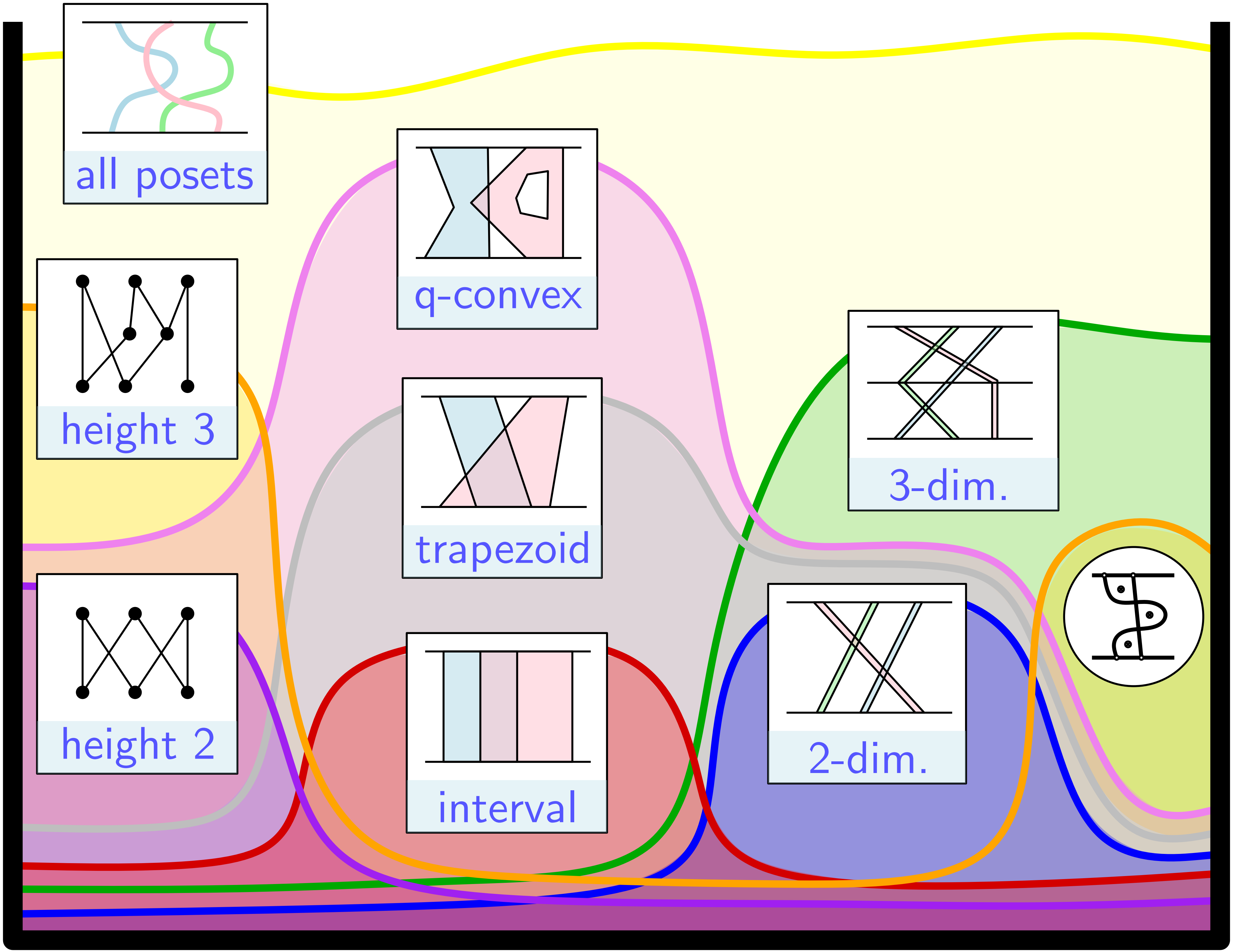

all posets
q-convex
trapezoid
height 2
interval
2-dim.

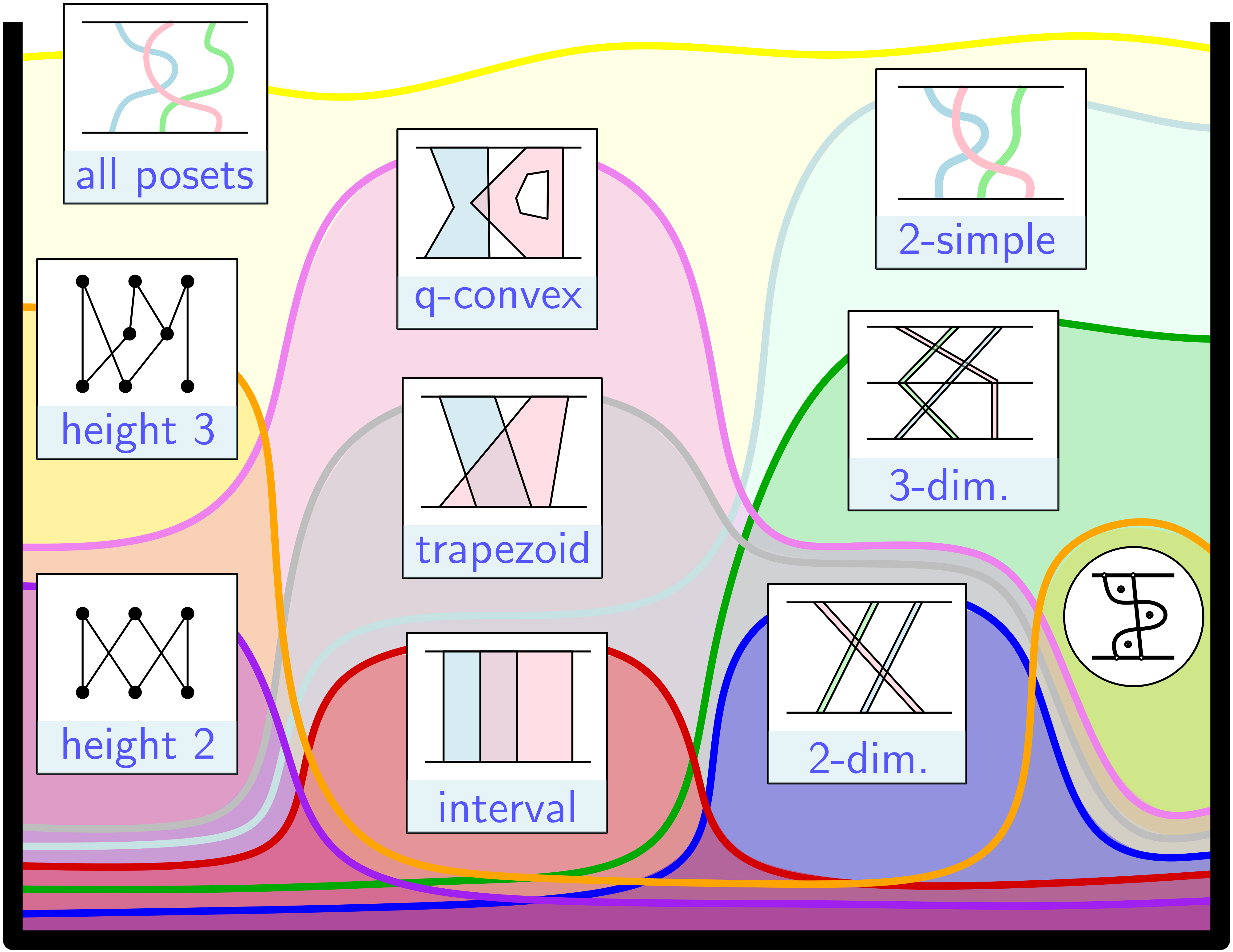

all posets
q-convex
height 3
trapezoid
height 2
interval
2-dim.

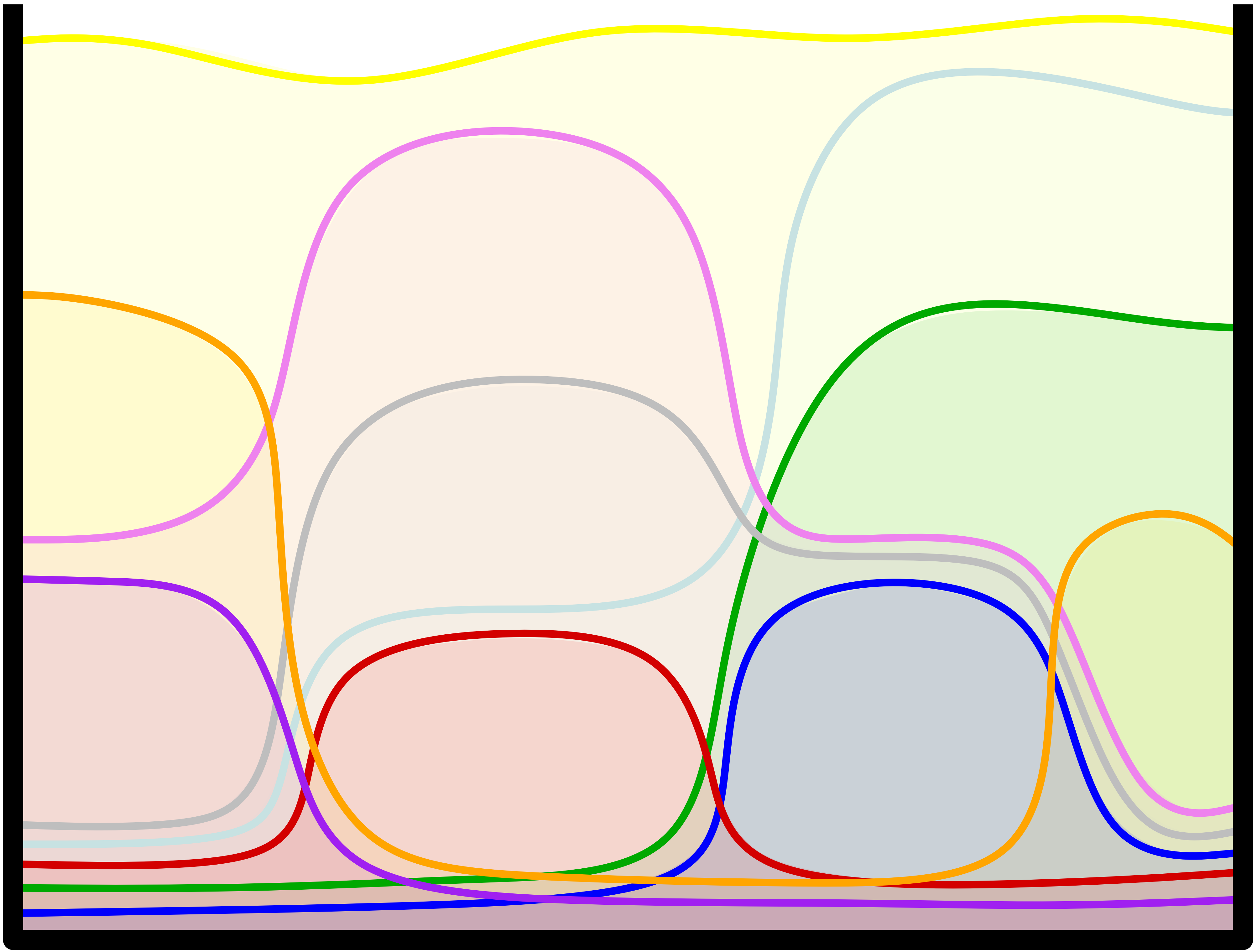

all posets
q-convex
height 3
trapezoid
3-dim.
height 2
2-dim.
interval

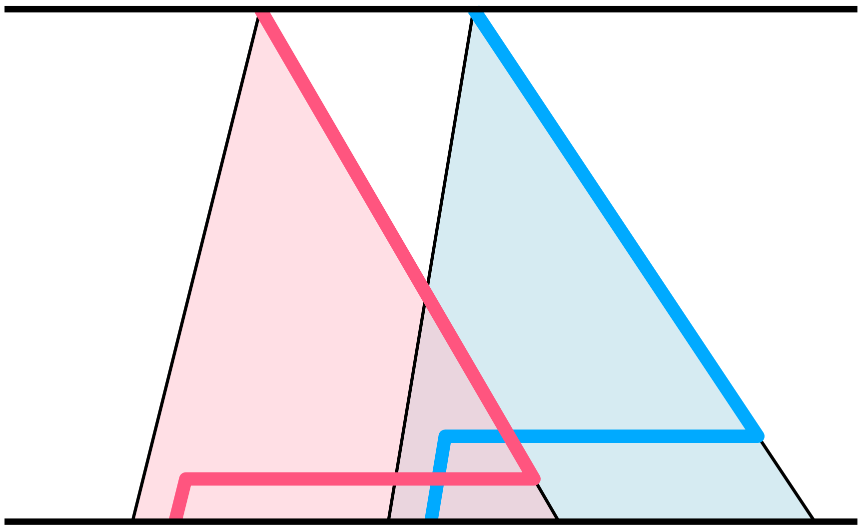
Lem. There exists a 3-dimensional poset of height 3 that "is not quasi-convex".




all posets
q-convex
height 3
trapezoid
3-dim.
height 2
interval
2-dim.

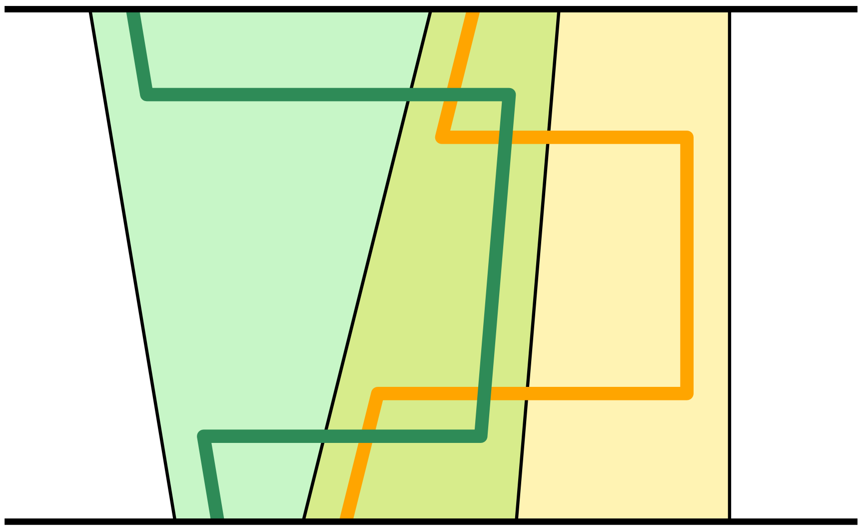

all posets
height 3
q-convex
2-simple
trapezoid
3-dim.
height 2
interval
2-dim.





triangles “are” 2-simple

(and thus also intervals)



trapezoids “are” 4-simple

Open Questions

- on-line coloring for convex sets with $o(\omega^3)$ colors ?
- on-line coloring 2-simple with polynomially many colors ?
- trapezoids $\not\subseteq$ 2-simple ?
- recognition of quasi-convex posets ?