Colorings of planar graphs

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Abstract

We will consider some vertex coloring concepts and discuss results especially for planar graphs.

Let $G = (V, E)$ a simple graph. For every vertex $v \in V(G)$ let $L(v)$ be a list of available colors. The collection of all lists is called a *list assignment*. A *k-assignment* is a list assignment where $|L(v)| = k$ for all $v \in V(G)$. The graph G is *L-colorable* if we can choose a color $\varphi(v)$ for every vertex from its list $L(v)$ such that $\varphi(v) \neq \varphi(w)$ for all $vw \in E(G)$. Moreover, G is *k-list colorable* if G is L-colorable for every k-assignment. It is well-known that every planar graph is 5-list colorable but there are planar graphs which are not 4-list colorable. There arises the question which additional properties of the list assignment can guarantee a 4-list coloring of planar graphs. We will consider two ideas dealing with intersections of lists.

A list assignment L for a graph $G = (V, E)$ is called a *$(k; c)$-assignment* if $|L(v)| = k$ for all $v \in V(G)$ and $|L(v) \cap L(w)| \leq c$ for all edges $vw \in E(G)$. A graph G is *$(k, b; c)$-list colorable* if for every possible $(k; c)$-assignment we can choose color sets $C(v) \subseteq L(v)$ such that $|C(v)| = b$ for all $v \in V(G)$ and $C(v) \cap C(w) = \emptyset$ for all edges $vw \in E(G)$. Beside some results there is still in open problem.

A k-assignment L satisfying $|\bigcap_{v \in V(G)} L(v)| \geq t$ is called a *t-common k-assignment*. Results on this topic will be interpreted as results for partitions of the vertex set.

Another interesting coloring concept asking on special 4-colorings of planar graphs are S-k-colorings introduced by Jiang and Zhu in 2020. The idea is that every (oriented) edge $e = vw$ has an assigned permutation $\sigma(e)$ from a set S. A vertex coloring $\varphi : V \to \{1, 2, \ldots, k\}$ is called an *S-k-coloring* if for every edge $e = vw$ it holds $\sigma(e)\varphi(v) \neq \varphi(w)$. This concept generalizes several other concepts, e.g. proper colorings, complex colorings, colorings of signed graphs and DP-colorings. For planar graphs, results on S-4-colorability are given where $S \subseteq S_4$ and S_4 is the set of all permutations π of $\{1, 2, 3, 4\}$.