Colorings of planar graphs
Margit Voigt
University of Applied Sciences Dresden, Germany

Abstract
We will consider some vertex coloring concepts and discuss results especially for planar graphs.

Let $G = (V, E)$ a simple graph. For every vertex $v \in V(G)$ let $L(v)$ be a list of available colors. The collection of all lists is called a list assignment. A $k$-assignment is a list assignment where $|L(v)| = k$ for all $v \in V(G)$. The graph $G$ is $L$-colorable if we can choose a color $\varphi(v)$ for every vertex from its list $L(v)$ such that $\varphi(v) \neq \varphi(w)$ for all $vw \in E(G)$. Moreover, $G$ is $k$-list colorable if $G$ is $L$-colorable for every $k$-assignment. It is well-known that every planar graph is 5-list colorable but there are planar graphs which are not 4-list colorable. There arises the question which additional properties of the list assignment can guarantee a 4-list coloring of planar graphs. We will consider two ideas dealing with intersections of lists.

A list assignment $L$ for a graph $G = (V, E)$ is called a $(k; c)$-assignment if $|L(v)| = k$ for all $v \in V(G)$ and $|L(v) \cap L(w)| \leq c$ for all edges $vw \in E(G)$. A graph $G$ is $(k; b; c)$-list colorable if for every possible $(k; c)$-assignment we can choose color sets $C(v) \subseteq L(v)$ such that $|C(v)| = b$ for all $v \in V(G)$ and $C(v) \cap C(w) = \emptyset$ for all edges $vw \in E(G)$. Beside some results there is still in open problem.

A $k$-assignment $L$ satisfying $|\bigcap_{v \in V(G)} L(v)| \geq t$ is called a $t$-common $k$-assignment. Results on this topic will be interpreted as results for partitions of the vertex set.

Another interesting coloring concept asking on special 4-colorings of planar graphs are $S$-$k$-colorings introduced by Jiang and Zhu in 2020. The idea is that every (oriented) edge $e = vw$ has an assigned permutation $\sigma(e)$ from a set $S$. A vertex coloring $\varphi : V \rightarrow \{1, 2, \ldots, k\}$ is called an $S$-$k$-coloring if for every edge $e = vw$ it holds $\sigma(e) \varphi(v) \neq \varphi(w)$. This concept generalizes several other concepts, e.g. proper colorings, complex colorings, colorings of signed graphs and DP-colorings. For planar graphs, results on $S$-4-colorability are given where $S \subseteq S_4$ and $S_4$ is the set of all permutations $\pi$ of $\{1, 2, 3, 4\}$. 