Exercise 1: Consider the following sets of real numbers: \( A = [-4, -1], B = \{-3, -2, 0, 3\} \) and \( C = [-2, 4] \).

(a) Determine the sets \( A \cup B, A \cap B \) and \((A \setminus C) \cup (C \setminus A)\).

(b) Find a maximal set \( M \) satisfying \((M \setminus A) \subseteq (B \cup C)\).

Exercise 2: Solve the following inequality and equality for \( x \):

(a) \((x - 5)^3(x + 1) \geq 0\),

(b) \(|x| = x^3 + 2x^2 - 3x\).

Exercise 3: Write each of the following sets as a union of intervals:

\[
U = \{x \in \mathbb{R} : ((x - 4)^2 - 10)^2 \geq 36\},
\]

\[
V = \{x \in \mathbb{R} : |2x + 6| + |2x - 6| - |x + 1| - |x - 1| > 8\}.
\]

Exercise 4: Evaluate the following sums:

(a) \(\sum_{n=7}^{42} \left(\frac{1}{3}\right)^n\),

(b) \(\sum_{m=-1}^{8} (n + 1)^3\) for \( n \in \mathbb{N} \),

(c) \(\sum_{\mu=0}^{1} \sum_{\nu=2}^{4} \frac{1}{\mu + \nu^2}\).

Exercise 5:

(a) Change the indices of the summations in the following expression so that the expression may be written with only one summation sign.

\[
\sum_{k=2}^{21} (k - 1)^2 + \sum_{\ell=-2}^{19} 2(\ell + 3) + \sum_{m=10}^{31} 1.
\]

Then evaluate the sum with the help of some results from the lecture.

(b) Prove the following identity for all \( a, b \in \mathbb{R} \) and \( n \in \mathbb{N} \) by means of changing indices:

\[
(a - b) \sum_{k=0}^{n} a^k b^{n-k} = a^{n+1} - b^{n+1}.
\]