

Supplementary problems 3

1. Express the following in the form $a + bi$ for real numbers a and b :

$$\frac{1 + 2i}{3 - 4i},$$

$$(1 + i)^5,$$

$$i^{101}.$$

2. Draw the region of the complex plane defined by the inequalities $3 < |z - 3| < 5$ for $z \in \mathbb{C}$.
3. Solve over the complex numbers the equation $z^2 - z = -10$.
4. Prove that $\arctan(1) + \arctan(2) + \arctan(3) = \pi$. (Hint: Use the fact that for complex numbers w and z , $\text{Arg}(wz) = \text{Arg}(w) + \text{Arg}(z)$ (shifted to be in $(-\pi, \pi]$), and choose simple complex numbers whose arguments are $\arctan(1)$, $\arctan(2)$ and $\arctan(3)$, respectively).