Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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Karlsruhe, December 20, 2018

Name: ....

Exercise Sheet No. 10
Advanced Mathematics I

Exercise 46: Prove the following identities using the addition theorems for sine and cosine:

(a) $\cot(a+b) = \frac{\cot a \cot b - 1}{\cot a + \cot b}$,

(b) $\tan \frac{x}{2} = \frac{\sin x}{1+\cos x}, \quad x \in (-\pi, \pi)$.

Exercise 47: Let the function $f : \mathbb{C} \to \mathbb{C}$ be defined by

$$f(z) = \cos(z)\sin(z) - 1.$$

Find all $z \in \mathbb{C}$, for which $f(z) = 0$.

Exercise 48:

Find the real and the imaginary part of all complex numbers $z \in \mathbb{C}$, which satisfy the equation

$$\frac{e^{iz} - 1}{1 - 2e^{-iz}} = 1\,.$$

Exercise 49:

(a) Find the real and imaginary parts of the numbers $i^i, \ln(i), \cos(i)$ and $e^{i\sqrt{2}}$.

(b) Show that the equation

$$\ln(u^v) = v\ln(u) \qquad \text{for } u, v \in \mathbb{C}$$

is not always satisfied.

Exercise 50: The power a^x is given by $a^x = e^{x \cdot \ln a}$ for $a, x \in \mathbb{R}$, $a > 0$. For some fixed a prove that

(a) $(a^x)^y = a^{xy}$ for all $x, y \in \mathbb{R}$,

(b) a^x is strictly monotonically increasing if $a > 1$, and strictly monotonically decreasing if $0 < a < 1$.

For $a = 10$ the inverse function $f^{-1}(y) = \log_{10}(y)$ of $f(x) = 10^x$ is the logarithm with respect to the basis 10.

(c) How can one use the logarithm $\ln(x)$ to compute the value $\log_{10}(x)$?

(d) Prove that
$$\begin{aligned} \log_{10}(xy) &= \log_{10}(x) + \log_{10}(y) \quad &, \ x, y > 0\,, \\ \log_{10}(x^y) &= y\log_{10}(x) \quad &, \ x > 0,\ y \in \mathbb{R}\,. \end{aligned}$$

Due date: Your written solutions are due at 14:00 on Tuesday, **15 January, 2019**.
Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag6/edu/am12018w/en`