Exercise 46: Prove the following identities using the addition theorems for sine and cosine:

(a) \(\cot(a + b) = \frac{\cot a \cot b - 1}{\cot a + \cot b}\),

(b) \(\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}, \quad x \in (-\pi, \pi)\).

Exercise 47: Let the function \(f : \mathbb{C} \to \mathbb{C}\) be defined by

\[f(z) = \cos(z) \sin(z) - 1.\]

Find all \(z \in \mathbb{C}\), for which \(f(z) = 0\).

Exercise 48:

Find the real and the imaginary part of all complex numbers \(z \in \mathbb{C}\), which satisfy the equation

\[\frac{e^{iz} - 1}{1 - 2e^{-iz}} = 1.\]

Exercise 49:

(a) Find the real and imaginary parts of the numbers \(i^i, \ln(i), \cos(i)\) and \(e^{i\sqrt{2}}\).

(b) Show that the equation

\[\ln(u^v) = v \ln(u) \quad \text{for } u, v \in \mathbb{C}\]

is not always satisfied.

Exercise 50: The power \(a^x\) is given by \(a^x = e^{x \ln a}\) for \(a, x \in \mathbb{R}\), \(a > 0\). For some fixed \(a\) prove that

(a) \((a^x)^y = a^{xy}\) for all \(x, y \in \mathbb{R}\),

(b) \(a^x\) is strictly monotonically increasing if \(a > 1\), and strictly monotonically decreasing if \(0 < a < 1\).

For \(a = 10\) the inverse function \(f^{-1}(y) = \log_{10}(y)\) of \(f(x) = 10^x\) is the logarithm with respect to the basis 10.

(c) How can one use the logarithm \(\ln(x)\) to compute the value \(\log_{10}(x)\)?

(d) Prove that

\[
\begin{align*}
\log_{10}(xy) &= \log_{10}(x) + \log_{10}(y), \quad x, y > 0, \\
\log_{10}(x^y) &= y \log_{10}(x), \quad x > 0, \ y \in \mathbb{R}.
\end{align*}
\]

Due date: Your written solutions are due at 14:00 on Tuesday, 15 January, 2019.
Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

http://www.math.kit.edu/iag6/edu/am12018w/en