Exercise Sheet No. 13
Advanced Mathematics I

Exercise 61:
Use integration by parts in order to determine the following integrals

\[ a) \int x \sin(2x) \, dx, \quad b) \int_0^{\pi/4} \frac{x}{\cos^2(x)} \, dx. \]

Exercise 62:
Find antiderivatives of the functions in (a),(b) and (c). Find the derivative of the function in (d).

(a) \( f: (1, \infty) \to \mathbb{R}, \quad f(x) = \frac{1}{x \ln(x) \ln(\ln x)} \)

(b) \( g: (0, \infty) \to \mathbb{R}, \quad g(x) = \left( \frac{\ln x}{x} \right)^2 \)

(c) \( h: \mathbb{R} \to \mathbb{R}, \quad h(x) = \frac{xe^{\arctan x}}{1 + x^2} \)

(d) \( k: \mathbb{R} \to \mathbb{R}, \quad k(x) = \int_{\cos(x)}^x t^2 e^t \, dt. \)

Exercise 63:
Find solutions for the initial value problems:

(a) \( u' = e^{-u} \cos x, \quad u(0) = 1, \quad (b) uu' + (1 + u^2) \sin x = 0, \quad u(0) = 1. \)

Exercise 64:
Solve the following differential equations:

(a) \( y'(x) - e^{-x} + y(x) - xy'(x) = xy(x), \quad y(0) = 1 \)

(b) \( x - y^2(x) + 2xy(x)y'(x) = 0 \text{ for } x > 0, \quad y(1) = 1. \)  \( \text{Hint: Substitute } z(x) = y^2(x). \)

Exercise 65: Solve the initial value problem

\[ y^3(x) - x^2 + xy^2(x)y'(x) = 0, \quad y(1) = 1. \]

Due date: Your written solutions are due at 14:00 on Tuesday, 5 February, 2019.
Please submit them at the beginning of the problem session.
Website: For detailed information regarding this course visit the following web page:
http://www.math.kit.edu/iag6/edu/am12018w/en