

31	32	33	34	35	Σ

Exercise Sheet No. 7 Advanced Mathematics I

Exercise 31:

(a) Compute the following limits:

$$(i) \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 4}}{\sqrt{x^2 + 9}}, \quad (ii) \lim_{x \rightarrow \infty} x^\alpha (\sqrt{x+1} - \sqrt{x}), \quad \alpha \in \mathbb{R}.$$

(b) Find $\lim_{x \rightarrow x_0} f(x)$ for the following functions f at x_0 :

$$(i) f(x) = \frac{x-2}{x^2-4} \text{ for } x > 2, \quad x_0 = 2, \quad (ii) f(x) = \frac{\sqrt[4]{x}-1}{\sqrt[3]{x}-1} \text{ for } x > 1, \quad x_0 = 1.$$

Exercise 32:

(a) Determine whether the following sets are open, closed or compact. Justify your answers.

$$M_1 = [-1, 42] \subset \mathbb{R}, \quad M_2 = (-1, 42] \subset \mathbb{R}, \quad M_3 = \{z \in \mathbb{C} : -1 \leq \text{Im } z \leq 1\} \subset \mathbb{C},$$

$$M_4 = \left\{ z \in \mathbb{C} : \text{Im } z \geq 0, \left| z - \frac{1}{2} \right| \leq \frac{1}{2} \right\} \setminus \{0\} \subset \mathbb{C}.$$

(b) Show that $z = i$ is in the boundary of M_3 and $z = \frac{1}{4}$ is an interior point of M_4 .

Exercise 33:

Consider the following polynomial p and function f :

$$p(x) = x^5 - 8x^2 + 4, \quad f(x) = |p(x)|, \quad x \in \mathbb{R}.$$

(a) Justify why the function must have a minimum and maximum value on $[-2, 2]$. Let the minimum be attained at x_- , and show $f(x_-) \leq 4$.

(b) Show that $f(x) \geq 4$ for all $|x| \geq 2$.

(c) Does f defined on \mathbb{R} have a minimum?

Exercise 34: How many solutions does the following equation have in the interval $I = [-4, 2]$?

$$x^5 + 8x^4 + 11x^3 = 25x^2 + 26x + 5$$

Justify your answer.

Hint: Formulate the equation in the form $f(x) = 0$ and evaluate f at the points $x = -4, -2, -\frac{1}{2}, 1$ and 2 .

Exercise 35: Show that for any positive constants a, b, c the equation

$$\frac{(a+b)x + a - b}{x^2 - 1} + \frac{c}{x - 2} = 1$$

has solutions in the intervals $[-1, 1]$ and $[1, 2]$, respectively.

Due date: Your written solutions are due at 14:00 on Tuesday, **11 December, 2018**.

Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page: