Exercise 26: Consider the polynomial \( f : \mathbb{R} \to \mathbb{R} \) given by \( f(x) := \frac{1}{8} x^3 + \frac{3}{8} x^2 - \frac{9}{8} x + \frac{5}{8} \).

(a) Express \( f(x) \) in terms of \( x - 1 \) and \( x + 3 \). Use this representation to discuss the behavior of \( f \) on the intervals \([-5, \infty)\) and \((-\infty, 3]\). Do not use the derivative \( f' \) for this discussion.

(b) Use a sketch of \( f \) to find intervals on which \( f \) has an inverse function. Also sketch the inverse on these intervals.

Exercise 27: Decide whether the functions are injective, surjective or bijective. If the function is bijective, then find the inverse. Justify your answer.

(a) \( f : \mathbb{R} \to \mathbb{R}_{\geq 0}, f(x) = |x| \)

(b) \( g : \mathbb{N} \to \mathbb{N}, g(n) = n + 1 \)

(c) \( h : \mathbb{N}_0 \to \mathbb{Z}, h(n) = \begin{cases} \frac{n}{2}, & \text{for } n \text{ even} \\ -\frac{n+1}{2}, & \text{for } n \text{ odd} \end{cases} \)

Exercise 28:
Using the sequence definition of continuity, find \( w \in \mathbb{R} \) so that the function
\[
\begin{align*}
f(x) &= \frac{x\sqrt{x} - 1}{\sqrt{x} - 1}, & x > 0, x \neq 1,
\end{align*}
\]
is continuous if we set \( f(1) := w \).

Exercise 29:
The function \( f : \mathbb{R} \setminus \{-2\} \to \mathbb{R} \) is defined as
\[
f(x) = \begin{cases} \frac{5x \cdot |x - 3|}{x^2 - x - 6}, & x \in \mathbb{R} \setminus \{-2, 1, 3\} \\ y_1 & x = 1 \\ y_2 & x = 3 \end{cases}
\]
Is it possible for \( f \) to be continuous at \( x = 1 \) and \( x = 3 \) with a suitable choice of \( y_1, y_2 \)? Give the appropriate values, or show that none exist.

Exercise 30:
Prove the following functions are Lipschitz-continuous and find the respective Lipschitz constant.

(a) \( f(x) = \sqrt{1 + 4x} \), \( D = [0, 4) \)

(b) \( f(x) = x^2 + 4x - 1 \), \( D = (-4, 3) \)

(c) \( f(x) = \sqrt{2x^2 + 1} \), \( D = [-2, 1] \)

Due date: Your written solutions are due at 14:00 on Tuesday, 4 December, 2018.
Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:
http://www.math.kit.edu/iag6/edu/am12018w/en