Problem sheet 11

Problems will be discussed at the problem class on **January 25, 2017**.

Problem 22.
Consider the set of ranges $\mathcal{F} = \{I \cup J \mid I, J \text{ interval in } \mathbb{R}, I \cap J = \emptyset\}$ on the real line. For each $k \geq 2$ calculate

$$m(k) = \min\{m \mid \chi_k(X, \mathcal{F}_m) \leq k \text{ for each finite set } X \subset \mathbb{R}\} \text{ and}$$

$$c(k) = \max\{\chi_k(X, \mathcal{F}_k) \mid X \subset \mathbb{R} \text{ finite set}\}.$$

Problem 23.
Given a finite family Y of subsets of some set V the *coverage* $p(v)$ of a point $v \in V$ is the number of elements of Y containing v. Let $p_{\max} = \max_{p \in V} c(p)$ and $p_{\min} = \min_{p \in V} c(p)$.

(a) Let Y be a finite family of intervals in $\mathbb{R}$. Prove that the intervals in Y can be colored using at most $p_{\max}$ colors such that intersecting intervals receive distinct colors.

(b) Let Y be a finite family of arcs on some circle. Prove that the arcs in Y can be colored using at most $p_{\max} + p_{\min}$ colors such that intersecting arcs receive distinct colors.

Puzzle 11.
What is the smallest number of circles in $\mathbb{R}^2$ covering the points of the regular 10×10 square grid? The circles may have different radii and a point is covered if it lies on a circle.