Problem sheet 13

Problems will be discussed at the problem class on **February 8, 2017**.

Problem 26.
For a set C of clothes of length at most 1 let $R^{\text{OFF}}(C)$ denote the smallest number of racks of unit length that can carry all clothes from C. The approximation ratio of an algorithm $\mathcal{A}$ for hanging laundry *online* is given by

$$\sup_{\substack{\text{C sequence of clothes}\\ \text{of length at most 1}}} \frac{\#racks\ used\ by\ \mathcal{A}\ for\ C}{R^{\text{OFF}}(C)}.$$

Prove that the best approximation ratio for hanging laundry online is between $\frac{4}{3}$ and 2.

Problem 27.
Let $\mathcal{F}$ denote the set of negative quadrants in $\mathbb{R}^2$ (translates of $\{(x, y) \in \mathbb{R}^2 \mid x, y \leq 0\}$), let $\mathcal{H}$ denote the set of left unbounded intervals in $\mathbb{R}$ (translates of $\{x \in \mathbb{R} \mid x \leq 0\}$), and let $\mathcal{C}_k = \{(X, \mathcal{H}_k) \mid X \text{ point set in } \mathbb{R}\}$, $k \geq 2$.

(a) Prove that for each point set $Y \subset \mathbb{R}^2$ we have $\chi_k(Y, \mathcal{F}_k) \leq \chi_k^{\text{ON}}(\mathcal{C}_k)$.

(b) For each $k \geq 2$ calculate $\chi_k^{\text{ON}}(\mathcal{C}_k)$.

Puzzle 13.
What is the maximal number of pieces you may obtain by slicing a donut with 3 plane cuts without rearranging the pieces in between?

Puzzle 14.
What is (most likely) the answer to this puzzle?