

**Problem sheet 3**

Problems will be discussed at the problem class on **November 16, 2016**.

**Problem 5.**

Find for all  $c > 0$  and all  $f > 1$  a connected graph  $G = G(c, f)$  with the properties that

$$\frac{|E(G)|}{|V(G)|} \leq f \quad \text{and} \quad \text{cr}(G) \geq c.$$

**Problem 6.**

Prove that  $I(3, 4) = 7$  and  $I(5, 6) = 14$ .

**Problem 7.**

For a fixed point set  $P$  and a positive integer  $k$  we call a line in the plane a *k-big line* if it contains at least  $k$  points from  $P$ . Let  $B_k(P)$  denote the number of  $k$ -big lines defined by  $P$  and  $B_k(n)$  the maximum of  $B_k(P)$  over all point sets  $P$  with  $|P| = n$ .

Prove that there is a constant  $c > 0$  such that for every  $n$  and every  $k$  with  $2 \leq k \leq \sqrt{n}$  we have

$$B_k(n) \leq c \frac{n^2}{k^3}.$$

**Puzzle 3.**

What is the largest  $k$  such that for every convex  $k$ -gon  $P$  one can place two copies  $P_1$  and  $P_2$  of  $P$  on top of each other (after rotation and/or translation) such that the resulting convex hull has  $2k$  corners, alternating between corners of  $P_1$  and corners of  $P_2$ ?