

# Problem sheet 1

Due Monday, April 23th at 17:30.

## Question 1

Let  $S$  be a set of  $n$  points in  $\mathbb{R}^2$  such that the distance between any pair of points is at most 1. Prove that there are at most  $\lfloor \frac{n^2}{3} \rfloor$  pairs of points in  $S$  whose distance is greater than  $\frac{1}{\sqrt{2}}$ .

## Question 2

Let  $f(n)$  denote the largest number of edges among all triangle-free graphs on  $n$  vertices that are non-bipartite. Prove for all  $n \geq 5$  that

$$f(n) = \text{ex}(n - 1, K_3) + 1.$$

## Question 3

Prove that  $\alpha(G) \geq \sum_v \frac{1}{d(v)+1}$ . Deduce Turán's theorem.

(**Hint:** Start from considering a random ordering of the vertices of  $G$ )

## Question 4

- (a) Prove that each  $k$ -edge-connected graph contains each tree on  $k + 1$  vertices.
- (b) Prove that each graph of girth at least  $k + 1$  and minimum degree  $d$  contains each tree of maximum degree  $d$  on  $k$  vertices as an induced subgraph.